Homework 9, due October 2

- (1) Construct a finite field with 9 elements and write down the addition and multiplication tables.
- (2) Let $GF(16) = \mathbb{Z}_2[x]/(x^4 + x + 1)$. Find a generator for the multiplicative group of GF(16). Write each element of GF(16) as a binary number, as a polynomial, and (except for 0) as a power of a generator.
- (3) If y = abcd in binary in GF(16) so that $y = ax^3 + bx^2 + cx + d$, find a formula for y^2 in binary. Find a formula for y^{14} in binary. If z = efgh, find a formula for yz. (You will probably want to use a computer algebra system. Note that in this field a coefficient such as a is either 0 or 1, and $a^2 = a$ and 2a = 0.)
- (4) In $GF(256) = \mathbb{Z}_2[x]/(x^8 + x^4 + x^3 + x + 1)$, calculate the following.
 - (a) 11110000 + 01011100
 - (b) 00001000 · 00010111
 - (c) 00111010 · 00010111
 - (d) 00000100^3
 - (e) 0000010^9
 - (f) 00000011^{-1}