

New Time Reversal Parities and Optimal Control of Dielectrics for Free Energy Manipulation

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Abstract: A dielectric's ultra-wide band *time-reversal spectrum* dictates optimal control of “in-and-out” real-time energy flows in dispersive media.

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1. Introduction

Traditional real-time notions of energy in dispersive dielectrics are typically ambiguous [1]. Ambiguity arises when a notion depends on details of microscopic models of media, rather than only on properties directly specifying the relevant macroscopic Maxwell equations. Worse, Landau and Lifshitz claimed that “...in the general case of arbitrary dispersion, the electromagnetic energy cannot be rationally defined as a thermodynamic quantity. This is because the presence of dispersion in general signifies a dissipation in energy...” [2]. Lacking thermodynamic interpretations in the dissipative case, they offered Brillouin's [3] unique narrow-bandwidth, *cycle-averaged* notion of energy for “dispersive but lossless” dielectrics [4]. Despite claims we show there are macroscopically-relevant, ultra-broadband notions of energy in arbitrarily dispersive-dissipative dielectrics with precise thermodynamic interpretation: they are *free energies* [5]. We also show the main effect of dissipation—multiple free energies—arises from a nontrivial *time-reversal spectrum* of the medium. The non-trivial time-reversal spectrum of any real medium is intimately connected to the non-trivial optimal control of real-time energy flow in and out of optical media.

A precise, broad-band, macroscopically relevant, notion of energy for arbitrary dissipative dielectrics was recently introduced. It is an instantaneous generalization of the Brillouin idea: at each instant during stimulation of a passive dielectric by an electromagnetic pulse, there is an unambiguous maximum energy that is subsequently recoverable from the medium [6]. This *recoverable energy* is extracted by an optimal “future” electromagnetic field starting from the instant considered. This instantaneous notion of electromagnetic energy in dispersive media depends only on the current dynamic macroscopic *state* of the medium. Recently it has also been shown that any given state of a linear, passive dielectric can be created in a unique, energetically minimal manner [7]. Both of these real-time, macroscopically-relevant notions of energy—the maximum energy extractable from a dispersive dielectric in any given state, and the minimal energy that could be imbued in it to produce the dielectric's current state—are independent of microscopic model. These “recoverable” and “creation” energies, U_+ and U_- , have recently been used to describe energetic features of “slow” and “fast” light, and to describe the energetics of optical regimes such as EIT where traditional notions of energy fail to have physical interpretation [7]. (See [8] and references for classical and new ideas about slow and fast light.) Here we relate the maximal and minimal free energies U_- and U_+ to the medium's *time-reversal spectrum*, giving rise to optimal energy injection and extraction fields. These turn out to be “orthogonal” under the medium's work function: superpositions of such fields act perfectly independently in the medium, i.e. giving rise to no interactions between these optimal field components via the medium interaction mechanism.

2. Time Reversal Spectrum and its Properties

For linear media, there is a class of “past” electromagnetic excitation fields $E_-(t)$ (vanishing after time $t = 0$) distinguished by the following property: when the medium is excited up until $t = 0$ by a past field $E_-(t)$ in this class, the medium is subsequently de-excited optimally—with maximal energy recovered from the dielectric—by a special

“future” field $E_+(t)$ (vanishing before $t = 0$) that is simply a time-reversed, dilated version of the original past field:

$$E_+(t) = \lambda E_-(-t). \quad (1)$$

Special past-future field pairs $(E_-(t), E_+(t))$ satisfying (1), $E_+(t)$ being optimal at extracting energy, expose various instantaneous energetics of linear dielectrics:

1. Only certain values of the dilation λ arise. Consistent with dissipation, these *time-reversal eigenvalues* lie in the interval $[-1, 1]$, and characterize a given medium.
2. The class of fields $E_-(t)$ satisfying (1) (with $E_+(t)$ optimal at extracting energy) are *complete*: Any medium state is obtainable by linear superposition in the class, and the class forms then a state-space *basis*.
3. The basis above is *preferred*: By design, $E_+(t)$ in (1) is optimal at extracting energy *from* the dielectric in the state produced by $E_-(t)$. But, fact, $E_-(t)$ satisfying (1) is itself optimal at infusing energy *into* the dielectric, the same true of superpositions of past fields satisfying (1). The last holds because of the following:
4. The *eigenspaces* of field excitations $E_-(t)$ satisfying (1) are orthogonal with respect to the medium’s work function: the work $W[E_-]$ performed on the dielectric by a superposition $E_- = c_1 E_{-\lambda_1} + c_2 E_{-\lambda_2}$ of energetically optimal past fields $E_{-\lambda_1}$ and $E_{-\lambda_2}$ with time-reversal eigenvalues $\lambda_1 \neq \lambda_2$ is the sum of the work performed in the dielectric by each individually, no constructive or destructive interference arising. That is,

$$W[c_1 E_{-\lambda_1} + c_2 E_{-\lambda_2}] = c_1^2 W[E_{-\lambda_1}] + c_2^2 W[E_{-\lambda_2}]. \quad (2)$$

5. Because of (2), the relationship of optimal energy infusion fields E_- to U_- , and that of optimal energy extraction fields E_+ to U_+ , these maximal and minimal free energies are diagonal quadratic forms in the preferred basis.
6. Two terms in the diagonal quadratic forms for both U_+ and U_- always arise, corresponding to time-reversal eigenvalues $\lambda = -1$ and $\lambda = +1$. These are identified as *en-masse* kinetic and potential energy, since they are forms in excitations that are momentum, respectively, position-like—i.e. odd and even—under time-reversal (1).
7. The quadratic forms of U_- and U_+ are the same except for time-reversal eigenvalues with $|\lambda| < 1$. The associated $|\lambda| < 1$ -energies allowing U_- and U_+ to be distinct give rise to the idea of “irreversible energies” and to the idea of a class of objects (field excitations) sporting time-reversal parities other than sign change.

Many of the listed properties of the time-reversal spectrum of dissipative macroscopic media are to be contrasted with the time-reversal properties of the *microscopic* physical quantities of electromagnetism. [9]

We show how the time-reversal spectrum of real dispersive media can be used to optimally control energy flow in such media.

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