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Coupling of Dirichlet-to-Neumann boundary condition and finite difference methods in curvilinear coordinates for multiple scattering

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ABSTRACT

The applicability of the Dirichlet-to-Neumann technique coupled with finite difference methods is enhanced by extending it to multiple scattering from obstacles of arbitrary shape. The original boundary value problem (BVP) for the multiple scattering problem is reformulated as an interface BVP. A heterogenous medium with variable physical properties in the vicinity of the obstacles is considered. A rigorous proof of the equivalence between these two problems for smooth interfaces in two and three dimensions for any finite number of obstacles is given. The problem is written in terms of generalized curvilinear coordinates inside the computational region. Then, novel elliptic grids conforming to complex geometrical configurations of several two-dimensional obstacles are constructed and approximations of the scattered field supported by them are obtained. The numerical method developed is validated by comparing the approximate and exact far-field patterns for the scattering from two circular obstacles. In this case, for a second order finite difference scheme, a second order convergence of the numerical solution to the exact solution is easily verified.

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1. Introduction

Analytical solutions for wave scattering problems from multiple complexly shaped obstacles are not possible to obtain in general. For this reason, early work was mainly performed on circular cylindrical and spherical obstacles using modal expansions of the scattered field. The construction of analytical techniques for multiple scattering continues to be an active field of research. Numerous works on multiple scattering from circular, elliptical cylinders, and spheres have recently appeared [1–3]. A major drawback of these methods is that they cannot be applied to more general scatterer geometries. The excellent book by Martin [4] reviews a variety of these analytical techniques and contains a large number of references.

Multiple scattering from scatterers of complex geometries requires the application of numerical techniques. Recent numerical work has been based on either finite difference, integral equation, or finite element methods. For instance, Sherer and Visbal [5] and Sherer and Scott [6] discussed multiple acoustic scattering from two and three circular cylinder configurations in two dimensions. For the approximations, they employed high-order compact finite difference methods on complex grids generated by overset-meshing procedures. Their numerical solution accurately approximates the analytical solution. Although, their technique has potential applications to scatterers of arbitrary shape, they only presented results for obstacles in the form of circular cylinders. Another attempt was made by Villamizar and Acosta [7] where an acoustic scattering problem from three complexly shaped obstacles was numerically solved. The approximation obtained for the acoustic pressure field is illustrated in Fig. 1 for a two-dimensional scatterer configuration. For this purpose, the authors used

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Although the computations of the multiple-DtN technique are performed in relatively small sub-domains, the linear system that results from the discretization of the continuous problem is not completely sparse. In fact, all the field values at the interface are also part of the unknowns. This may be a disadvantage when compared with the integral equation methods, such as Nyström or boundary elements, whose matrices are dense but their only unknowns are at the obstacle boundaries. However, when the properties of the medium change, the use of integral equation methods may result in comparable or larger linear systems than those obtained by employing the proposed method.

We are currently working on the implementation of the theoretical results found in this work to configurations of truly complex three-dimensional obstacles. One of the major challenges in doing this will be the extension of the grid generation technique to three-dimensional scatterer configurations. However, the fact that grids are independently generated in sub-domains containing a single obstacle will greatly simplify the process. In order to improve the rate of convergence and the computational efficiency, we plan to use higher order compact schemes [44] instead of our current second order method.

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