Outline Thermodynamics of Dispersion Local Free Energies Global Free Energies Designer Pulses Designer Media Summary L 000 00 00 00 00 00 00

Designer Media and Pulses for Optimally Long-Lived and Reversible Energy Storage

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Thermodynamics of Dispersion

What is Free Energy?

Free Energies and Irreversibility

Local Free Energies

The Free Energy & Loss Representation of Work

Global Free Energies

The Free Energy & Loss Representation of Work

Designer Pulses

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Media

Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

Summary

Local Free Energy Simulations: the EIT anomaly - (B) (E) (E) (E) (E)



What is free energy?

Gibb's Free energy: "...the greatest amount of ... work which can be obtained from a given quantity of a certain substance in a given initial state-without increasing its total volume or allowing heat to pass to or from external bodies, except such as at the close of the processes are left in their initial condition." –J. Willard Gibbs ¹

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pp.398,399, Dec. 1873

¹ A Method of Geometrical Representation of the Thermodynamic Properties of Substances by Means of Surfaces, Trans. Conn. Acad. II



- Axiom 1: A dynamical free energy is a state function.
- Axiom 2: The Second Law is "Work performed by a body must be performed at the expense of at least that much free energy." ²
- Result of Axioms–"First Law":

$$rac{dW_{ ext{on body}}}{dt} = rac{dF_{ ext{of body}}}{dt} + rac{dQ_{ ext{F loss}}}{dt} \geq rac{dF_{ ext{of body}}}{dt}$$

▶ Hindsight Axiom: Both F_{of body} and R_{F loss} := dQ_{F loss}/dt ≥ 0 are state functions.

² Clausius-Duhem inequality: V. Berti and G. Gentili, J. Non-Equilib. Thermodyn. 24, 154 (1999). ³ Chu, X.; Ross, J.; Hunt, P. M.; Hunt, K. L. C. J. Chem. Phys. 1993, 99,3444.



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Poynting's Conservation Law+First Law=Dissipation Law

Poynting's Conservation Law:

$$0 = \nabla \cdot \mathbf{S}(t) + \frac{\partial u_{\text{field}}(t)}{\partial t} + \frac{\partial u_{\text{int}}(t)}{\partial t}$$

► First Law:

$$u_{\mathrm{int}}(t) := \int_{-\infty}^{t} E(\tau) \dot{P}(\tau) d\tau = W(t) = F(t) + Q(t)$$

Poynting's Conservation Law+First Law:

$$0 \ge -R(t) = -rac{\partial Q(t)}{\partial t} = \nabla \cdot \mathbf{S}(t) + rac{\partial}{\partial t} (u_{ ext{field}}(t) + F(t))$$

$$rac{d}{dt}\mathsf{F}(\mathsf{t})\leq 0$$
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A Work Representation & a Local Free Energy

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A Work Representation & a Local Free Energy

$$W[E](t) = \int_{-\infty}^{t} E(\tau)\dot{P}(\tau)d\tau = \int_{-\infty}^{+\infty} \omega \operatorname{Im}\left[\chi\right](\omega) \left|\hat{E}_{t}\right|^{2}(\omega)d\omega$$
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$$= \underbrace{\frac{Q_{max}[E](t)}{\omega_{p}^{2}} \int_{-\infty}^{t} \dot{P}_{min}^{2}\left[E_{t}\right](\tau) d\tau}_{p} + \underbrace{\frac{\gamma_{p}}{\omega_{p}^{2}} \int_{t}^{+\infty} \dot{P}_{min}^{2}\left[E_{t}\right](\tau) d\tau}_{p};$$

 $P_{\min}[E] = \chi_{\min}E,$

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$$\widehat{P_{min}^{2}\left[E\right]} = \chi_{min}\hat{E}, \qquad |\chi_{min}(\omega)|^{2} = \frac{\omega_{p}^{2}}{\gamma_{p}} \frac{\operatorname{Im}[\chi](\omega)}{\omega}$$

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$$= \underbrace{\frac{Q_{\max}[E](t)}{\omega_{p}^{2}} \int_{-\infty}^{t} \dot{P}_{\min}^{2}\left[E\right](\tau) d\tau}_{P\min} + \underbrace{\frac{\gamma_{p}}{\omega_{p}^{2}} \int_{t}^{+\infty} \dot{P}_{\min}^{2}\left[E_{t}\right](\tau) d\tau}_{F\min};$$
$$\widehat{P_{\min}[E]} = \chi_{\min}\hat{E}, \qquad |\chi_{\min}(\omega)|^{2} = \frac{\omega_{p}^{2}}{\gamma_{p}} \frac{\operatorname{Im}[\chi](\omega)}{\omega} > 0$$

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$$\widehat{P_{\min}[E]} = \chi_{\min}\hat{E}, \ \chi_{\min} \in \mathcal{A}_{\chi}^{++}, \chi_{\min}(\omega)\chi_{\min}(-\omega) = \frac{\omega_{p}^{2}}{\gamma_{p}} \frac{\operatorname{Im}[\chi](\omega)}{\omega}$$

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The Extreme Local Free Energy Representations

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The Extreme Local Free Energy Representations

W[E](t)



The Extreme Local Free Energy Representations

 $W[E](t) = Q_{max}[E](t) + F_{min}[E](t)$

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= $\frac{\gamma_p}{\omega_p^2} \int_{-\infty}^{t} \dot{P}_{min}^2[E](\tau) d\tau + \frac{\gamma_p}{\omega_p^2} \int_{t}^{+\infty} \dot{P}_{min}^2[E_t](\tau) d\tau;$

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Standard R.H. Factorization Problem
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$$W[E](t)$$

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$$W[E](t) = Q_{min}[E](t) + F_{max}[E](t)$$

$$= \frac{\gamma_p}{\omega_p^2} \int_{-\infty}^{t} \dot{P}_{max}^2 [E](\tau) d\tau + \frac{\gamma_p}{\omega_p^2} \int_{t}^{+\infty} \dot{P}_{max}^2 [E_t](\tau) d\tau;$$

$$W[E](t) = Q_{max} [E](t) + F_{min} [E](t)$$

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Standard R.H. Factorization Problem
$$\chi_{min}(\omega) \in \mathcal{A}_{\chi}^+, 1/\chi_{min}(\omega) \in \mathcal{A}_{\chi}^+, \ \chi_{min}(\omega)\chi_{min}(-\omega) = \frac{\omega_p^2}{\gamma_p} \frac{\mathrm{Im}[\chi](\omega)}{\omega}.$$

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Generalized/Nonlocal R.H. Factorization Problem
$$\overline{\chi_{max}(\omega) \in \mathcal{A}_{\chi}^+, 1/\chi_{max}(\omega) \in \mathcal{A}_{\chi}^-, \ \chi_{max}(\omega)\chi_{max}(-\omega) = \frac{\omega_p^2}{\gamma_p} \frac{\mathrm{Im}[\chi](\omega)}{\omega}.$$

The Free Energy & Loss Representation of Work

A Work Representation & a *Bulk* Free Energy: Semi-Infinite Slab $z \in [0, +\infty)$

The Free Energy & Loss Representation of Work

A Work Representation & a *Bulk* Free Energy: Semi-Infinite Slab $z \in [0, +\infty)$

W[E](t)

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The Free Energy & Loss Representation of Work

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$$W[E](t) = \int_0^\infty \int_{-\infty}^t E(z,\tau) \dot{P}(z,\tau) d\tau dz$$

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$$W[E](t) = \int_0^\infty \int_{-\infty}^t E(z,\tau) \dot{P}(z,\tau) d\tau dz$$
$$= \int_{-\infty}^{+\infty} \omega \operatorname{Im}[\chi](\omega) \int_0^\infty \left| \widehat{E}_t(z,\omega) \right|^2 dz \, d\omega$$

The Free Energy & Loss Representation of Work

A Work Representation & a *Bulk* Free Energy: Semi-Infinite Slab $z \in [0, +\infty)$

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Propagation

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Propagation $\int_{-\infty}^{+\infty} c n(\omega) \left|\widehat{E}_{t}(0,\omega)\right|^{2} d\omega$

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Propagation $\int_{-\infty}^{+\infty} c \, n(\omega) \left|\hat{E}_{t}(0,\omega)\right|^{2} d\omega = c \int_{-\infty}^{+\infty} \left|N(\omega) \, \hat{E}_{t}(0,\omega)\right|^{2} d\omega$

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The Free Energy & Loss Representation of Work

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The Free Energy & Loss Representation of Work

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$$|N(\omega)|^2 = n(\omega) > 0$$

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$$\mathcal{W}[E](t) = \int_{0}^{\infty} \int_{-\infty}^{t} E(z,\tau)\dot{P}(z,\tau)d\tau dz$$

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$$N(\omega) \in \mathcal{A}_n^+, \qquad \qquad N(\omega)N(-\omega) = n(\omega)$$

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The Free Energy & Loss Representation of Work

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$$\mathcal{W}[E](t) = \int_{0}^{\infty} \int_{-\infty}^{t} E(z,\tau)\dot{P}(z,\tau)d\tau dz$$

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Propagation $\int_{-\infty}^{+\infty} c \, n(\omega) \left|\hat{E}_{t}(0,\omega)\right|^{2} d\omega = c \int_{-\infty}^{+\infty} \left|N(\omega) \, \hat{E}_{t}(0,\omega)\right|^{2} d\omega$

$$= c \int_{-\infty}^{+\infty} E_{min}^{2} \left[E_{t}(0,*)\right](\tau) d\tau; \, \widehat{E_{min}[E]} = N\widehat{E},$$

Standard R.H. Factorization Problem
 $N(\omega) \in \mathcal{A}_{n}^{+}, 1/N(\omega) \in \mathcal{A}_{n}^{+}, N(\omega)N(-\omega) = n(\omega)$

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The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

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The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

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The Free Energy & Loss Representation of Work

The Extreme *Bulk* Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

$$W[E](t) = c \int_{-\infty}^{t} E_{min}^{2} [E_{t}(0,*)](\tau) d\tau + c \int_{t}^{+\infty} E_{min}^{2} [E_{t}(0,*)](\tau) d\tau;$$

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The Free Energy & Loss Representation of Work

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The Free Energy & Loss Representation of Work

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The Free Energy & Loss Representation of Work

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The Free Energy & Loss Representation of Work

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The Free Energy & Loss Representation of Work

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The Free Energy & Loss Representation of Work

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$$W[E](t)$$

The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

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$$W[E](t) = c \int_{-\infty}^{t} E_{max}^{2}[E_{t}(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{max}^{2}[E_{t}(0,*)](\tau)d\tau;$$

The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

$$\begin{aligned} & Q_{max}[E](t) \qquad F_{min}[E](t) \\ & W[E](t) = c \int_{-\infty}^{t} E_{min}^{2} \left[E\left(0,*\right) \right](\tau) d\tau + c \int_{t}^{+\infty} E_{min}^{2} \left[E_{t}(0,*) \right](\tau) d\tau ; \\ & \widehat{E_{min}[E]} = N_{<} \widehat{E}, N_{<}(\omega) \in \mathcal{A}_{n}^{+}, 1/N_{<}(\omega) \in \mathcal{A}_{n}^{+}, N_{<}(\omega) N_{<}(-\omega) = n(\omega) \\ & Q_{min}[E](t) \\ & W[E](t) = c \int_{-\infty}^{t} E_{max}^{2} \left[E_{t}(0,*) \right](\tau) d\tau + c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}(0,*) \right](\tau) d\tau ; \end{aligned}$$
The Free Energy & Loss Representation of Work

The Extreme *Bulk* Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

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The Free Energy & Loss Representation of Work

The Extreme *Bulk* Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

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The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

$$Q_{max}[E](t) = c \int_{-\infty}^{t} E_{min}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{min}^{2}[E_{t}(0,*)](\tau)d\tau;$$

$$\widehat{E_{min}[E]} = N_{<}\widehat{E}, N_{<}(\omega) \in \mathcal{A}_{n}^{+}, 1/N_{<}(\omega) \in \mathcal{A}_{n}^{+}, N_{<}(\omega)N_{<}(-\omega) = n(\omega)$$

$$Q_{min}[E](t) = c \int_{-\infty}^{t} E_{max}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{max}^{2}[E_{t}(0,*)](\tau)d\tau;$$

$$\widehat{E_{max}[E]} = N_{>}\widehat{E},$$

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The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

$$Q_{max}[E](t) = c \int_{-\infty}^{t} E_{min}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{min}^{2}[E_{t}(0,*)](\tau)d\tau;$$

$$\widehat{E_{min}[E]} = N_{<}\widehat{E}, N_{<}(\omega) \in \mathcal{A}_{n}^{+}, 1/N_{<}(\omega) \in \mathcal{A}_{n}^{+}, N_{<}(\omega)N_{<}(-\omega) = n(\omega)$$

$$Q_{min}[E](t) = c \int_{-\infty}^{t} E_{max}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{max}^{2}[E_{t}(0,*)](\tau)d\tau;$$

$$\widehat{E_{max}[E]} = N_{>}\widehat{E}, \qquad |N_{>}(\omega)|^{2} = n(\omega)$$

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The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

$$Q_{max}[E](t) = c \int_{-\infty}^{t} E_{min}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{min}^{2}[E_{t}(0,*)](\tau)d\tau;$$

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$$Q_{min}[E](t) = c \int_{-\infty}^{t} E_{max}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{max}^{2}[E_{t}(0,*)](\tau)d\tau;$$

$$\widehat{E_{max}[E]} = N_{>}\widehat{E}, \qquad N_{>}(\omega)N_{>}(-\omega) = n(\omega)$$

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The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

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The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

$$Q_{max}[E](t) = c \int_{-\infty}^{t} E_{min}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{min}^{2}[E_{t}(0,*)](\tau)d\tau;$$

$$\widehat{E_{min}[E]} = N_{<}\widehat{E}, N_{<}(\omega) \in \mathcal{A}_{n}^{+}, 1/N_{<}(\omega) \in \mathcal{A}_{n}^{+}, N_{<}(\omega)N_{<}(-\omega) = n(\omega)$$

$$Q_{min}[E](t) = c \int_{-\infty}^{t} E_{max}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{max}^{2}[E_{t}(0,*)](\tau)d\tau;$$

$$\widetilde{E_{max}[E]} = N_{>}\widetilde{E}, N_{>}(\omega) \in \mathcal{A}_{n}^{+}, 1/N_{>}(\omega) \in \mathcal{A}_{n}^{-}, N_{>}(\omega)N_{>}(-\omega) = n(\omega)$$

The Free Energy & Loss Representation of Work

The Extreme Bulk Free Energy Representations: Semi-Infinite Slab $z \in [0, +\infty)$

$$Q_{max}[E](t) = c \int_{-\infty}^{t} E_{min}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{min}^{2}[E_{t}(0,*)](\tau)d\tau;$$

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$$Q_{min}[E](t) \qquad F_{max}[E](t)$$

$$W[E](t) = c \int_{-\infty}^{t} E_{max}^{2}[E(0,*)](\tau)d\tau + c \int_{t}^{+\infty} E_{max}^{2}[E_{t}(0,*)](\tau)d\tau;$$
Generalized/Nonlocal R.H. Factorization Problem
$$\widehat{E_{max}[E]} = N_{>}\hat{E}, N_{>}(\omega) \in \mathcal{A}_{n}^{+}, 1/N_{>}(\omega) \in \mathcal{A}_{n}^{-}, N_{>}(\omega)N_{>}(-\omega) = n(\omega)$$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

 $F_{min}[E](t)$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

 $F_{min}[E](t) := \max_{E_t^+} -\Delta_{[t,+\infty)} W[E_t^- + E_t^+]$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$F_{min}[E](t) := \max_{E_t^+} -\Delta_{[t,+\infty)} W[E_t^- + E_t^+]$$

= $W[E](t) - \min_{E_t^+} W[E_t^- + E_t^+](+\infty)$

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Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) := \max_{E_t^+} -\Delta_{[t,+\infty)} W[E_t^- + E_t^+]$$

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 $W[E_t^- + E_t^+](+\infty)$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

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 $W[E_t^- + E_t^+](+\infty) = c \int_{-\infty}^t E_{min}^2 \left[E_t^-(0,*) \right](\tau) d\tau + c \int_t^{+\infty} E_{min}^2 \left[\left(E_t^- + E_t^+ \right)(0,*) \right](\tau) d\tau$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) := \max_{E_t^+} -\Delta_{[t,+\infty)} W[E_t^- + E_t^+]$$

= $W[E](t) - \min_{E_t^+} W[E_t^- + E_t^+](+\infty)$
 $W[E_t^- + E_t^+](+\infty) = c \int_{-\infty}^t E_{min}^2 \left[E_t^-(0,*) \right](\tau) d\tau +$
 $c \int_t^{+\infty} E_{min}^2 \left[\left(E_t^- + E_t^+ \right)(0,*) \right](\tau) d\tau$
 $\ge c \int_{-\infty}^t E_{min}^2 \left[E_t^-(0,*) \right](\tau) d\tau$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$F_{min}[E](t) := \max_{E_t^+} -\Delta_{[t,+\infty)} W[E_t^- + E_t^+]$$

= $W[E](t) - \min_{E_t^+} W[E_t^- + E_t^+](+\infty)$
 $W[E_t^- + E_t^+](+\infty) = c \int_{-\infty}^t E_{min}^2 \left[E_t^-(0,*) \right](\tau) d\tau +$
 $c \int_{-\infty}^{+\infty} E_{min}^2 \left[\left(E_t^- + E_t^+ \right)(0,*) \right](\tau) d\tau$
 $\ge c \int_{-\infty}^t E_{min}^2 \left[E_t^-(0,*) \right](\tau) d\tau$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$F_{min}[E](t) := \max_{E_t^+} -\Delta_{[t,+\infty)} W[E_t^- + E_t^+]$$

= $W[E](t) - \min_{E_t^+} W[E_t^- + E_t^+](+\infty)$
 $W[E_t^- + E_t^+](+\infty) = c \int_{-\infty}^t E_{min}^2 \left[E_t^-(0,*) \right](\tau) d\tau +$
 $0 = c \int_t^{+\infty} E_{min}^2 \left[\left(E_t^- + E_t^+ \right)(0,*) \right](\tau) d\tau$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) := \max_{E_{t}^{+}} -\Delta_{[t,+\infty)} W[E_{t}^{-} + E_{t}^{+}]$$

= $W[E](t) - \min_{E_{t}^{+}} W[E_{t}^{-} + E_{t}^{+}](+\infty)$
 $W[E_{t}^{-} + E_{t}^{+}](+\infty) = c \int_{-\infty}^{t} E_{min}^{2} [E_{t}^{-}(0,*)](\tau) d\tau +$
 $0 = c \int_{t}^{+\infty} E_{min}^{2} [(E_{t}^{-} + E_{t}^{+})(0,*)](\tau) d\tau$
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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) := \max_{E_{t}^{+}} -\Delta_{[t,+\infty)} W[E_{t}^{-} + E_{t}^{+}]$$

$$= W[E](t) - \min_{E_{t}^{+}} W[E_{t}^{-} + E_{t}^{+}](+\infty)$$

$$W[E_{t}^{-} + E_{t}^{+}](+\infty) = c \int_{-\infty}^{t} E_{min}^{2} \left[E_{t}^{-}(0,*) \right](\tau) d\tau +$$

$$0 = c \int_{t}^{+\infty} E_{min}^{2} \left[\left(E_{t}^{-} + E_{t}^{+} \right)(0,*) \right](\tau) d\tau$$

$$\iff$$

$$0 = E_{min} \left[\left(E_{t}^{-} + E_{t}^{+} \right)(0,*) \right](\tau) \text{ for } \tau > t$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$



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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$



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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

 $E_t^+ \left[E_t^-(0,*) \right]$:

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

 $E_{t}^{+}\left[E_{t}^{-}(0,*)\right] : \mathcal{F}\left[E_{min}\right]\left[\left(E_{t}^{-}+E_{t}^{+}\right)(0,*)\right]$

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Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

 $E_{t}^{+}\left[E_{t}^{-}(0,*)\right] : \mathcal{F}\left[E_{min}\right]\left[\left(E_{t}^{-}+E_{t}^{+}\right)(0,*)\right] \in \mathcal{E}_{t}^{-}$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$E_{t}^{+}\left[E_{t}^{-}(0,*)\right] : \mathcal{F}\left[E_{min}\right]\left[\left(E_{t}^{-}+E_{t}^{+}\right)(0,*)\right] \in \mathcal{E}_{t}^{-} \qquad \Longleftrightarrow$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right) (0,*) \right] \ \in \mathcal{E}_t^- \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] \ + \ N_< \mathcal{F}\left[E_t^+(0,*) \right] \ \in \mathcal{E}_t^- \end{split}$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right)(0,*) \right] \in \mathcal{E}_t^- & \Longleftrightarrow \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] \in \mathcal{E}_t^- & \Longrightarrow \end{split}$$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right) (0,*) \right] \in \mathcal{E}_t^- & \iff \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] \in \mathcal{E}_t^- & \Longrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ P_t^+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 \end{split}$$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right) (0,*) \right] \in \mathcal{E}_t^- & \Longleftrightarrow \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] \in \mathcal{E}_t^- & \Longrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ P_t^+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \Longleftrightarrow \end{split}$$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right) (0,*) \right] \in \mathcal{E}_t^- & \Longleftrightarrow \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] \in \mathcal{E}_t^- & \Longrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ P_t^+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \Longleftrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 \end{split}$$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right) (0,*) \right] \in \mathcal{E}_t^- & \Longleftrightarrow \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] \in \mathcal{E}_t^- & \Longrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ P_t^+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \Longleftrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \Longleftrightarrow \end{split}$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right) (0,*) \right] \in \mathcal{E}_t^- & \iff \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] \in \mathcal{E}_t^- & \Longrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ P_t^+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \iff \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \iff \end{split}$$

$$E_{\boldsymbol{t}}^{+}(0,\tau) = -\mathcal{F}^{-1} \circ N_{<}^{-1} \circ P_{\boldsymbol{t}}^{+} \circ N_{<} \circ \mathcal{F}\left[E_{\boldsymbol{t}}^{-}(0,*)\right](\tau);$$

Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

$$\begin{split} E_t^+ \left[E_t^-(0,*) \right] &: \ \mathcal{F}\left[E_{min} \right] \left[\left(E_t^- + E_t^+ \right) (0,*) \right] \in \mathcal{E}_t^- & \iff \\ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] \in \mathcal{E}_t^- & \Longrightarrow \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ P_t^+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \iff \\ P_t^+ N_< \mathcal{F}\left[E_t^-(0,*) \right] &+ N_< \mathcal{F}\left[E_t^+(0,*) \right] = P_t^+ \mathcal{E}_t^- &= 0 & \iff \end{split}$$

$$\begin{split} E_{\boldsymbol{t}}^{+}(\boldsymbol{0},\tau) &= \qquad -\mathcal{F}^{-1} \circ N_{<}^{-1} \circ P_{\boldsymbol{t}}^{+} \circ N_{<} \circ \mathcal{F}\left[E_{\boldsymbol{t}}^{-}(\boldsymbol{0},*)\right](\tau); \\ P_{\boldsymbol{t}}^{+} &= \qquad \mathcal{F} \circ \Theta_{\boldsymbol{t}}^{+} \circ \mathcal{F}^{-1} \end{split}$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) = \max_{E_t^+} -\Delta_{[t,+\infty)} W[E_t^- + E_t^+];$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) = \max_{E_{t}^{+}} -\Delta_{[t,+\infty)} W[E_{t}^{-} + E_{t}^{+}];$$

$$E_{t}^{+}(0,\tau) = -\mathcal{F}^{-1} \circ N_{<}^{-1} \circ P_{t}^{+} \circ N_{<} \circ \mathcal{F}\left[E_{t}^{-}(0,*)\right](\tau);$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) = \max_{E_{t}^{+}} -\Delta_{[t,+\infty)} W[E_{t}^{-} + E_{t}^{+}];$$

$$\begin{split} E_t^+(0,\tau) &= \qquad -\mathcal{F}^{-1} \circ N_{<}^{-1} \circ P_t^+ \circ N_{<} \circ \mathcal{F}\left[E_t^-(0,*)\right](\tau);\\ P_t^+ &= \qquad \mathcal{F} \circ \Theta_t^+ \circ \mathcal{F}^{-1}; \end{split}$$

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Optimal Energy Extraction Pulses: Steering the Minimum Free Energy in Bulk

Designer Pulse for Optimal Energy Extraction: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{min}[E](t) = \max_{E_{t}^{+}} -\Delta_{[t,+\infty)} W[E_{t}^{-} + E_{t}^{+}];$$

$$\begin{split} E_t^+(0,\tau) &= & -\mathcal{F}^{-1} \circ N_{<}^{-1} \circ P_t^+ \circ N_{<} \circ \mathcal{F} \left[E_t^-(0,*) \right] (\tau); \\ P_t^+ &= & \mathcal{F} \circ \Theta_t^+ \circ \mathcal{F}^{-1}; \\ N_{<}(\omega) &= & \exp \circ P_0^+ \circ \log n(\omega). \end{split}$$
Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

 $F_{max}[E](t)$



Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-]$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}\left[E\right]\left(t\right) := \min_{G_t^- \in \sigma\left(E_t^-\right)} W[G_t^-] = c \int_t^{+\infty} E_{max}^2 \left[E_t^-(0,*)\right](\tau)$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-] = c \int_t^{+\infty} E_{max}^2 \left[E_t^-(0,*) \right](\tau)$$

 $W[G_t^-]$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-] = c \int_t^{+\infty} E_{max}^2 \left[E_t^-(0,*) \right](\tau)$$
$$W[G_t^-] = c \int_t^t E_{max}^2 \left[G_t^-(0,*) \right](\tau) d\tau +$$

$$c\int_{t}^{+\infty} E_{max}^{2}\left[G_{t}^{-}(0,*)\right](\tau)d\tau$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-] = c \int_t^{+\infty} E_{max}^2 \left[E_t^-(0,*) \right](\tau)$$
$$W[G_t^-] = c \int_t^t E_{max}^2 \left[G_t^-(0,*) \right](\tau) d\tau + c$$

$$c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0,*) \right] (\tau) d\tau$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}[E](t) := \min_{G_{t}^{-} \in \sigma(E_{t}^{-})} W[G_{t}^{-}] = c \int_{t}^{+\infty} E_{max}^{2} [E_{t}^{-}(0,*)](\tau)$$
$$W[G_{t}^{-}] = c \int_{-\infty}^{t} E_{max}^{2} [G_{t}^{-}(0,*)](\tau) d\tau + c \int_{t}^{+\infty} E_{max}^{2} [E_{t}^{-}(0,*)](\tau) d\tau + c \int_{t}^{+\infty} E_{max}^{2} [E_{t}^{-}(0,*)](\tau) d\tau$$
$$\geq c \int_{t}^{+\infty} E_{max}^{2} [E_{t}^{-}(0,*)](\tau) d\tau$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

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$$F_{max}[E](t) := \min_{G_{t}^{-} \in \sigma(E_{t}^{-})} W[G_{t}^{-}] = c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0,*) \right](\tau)$$
$$W[G_{t}^{-}] = c \int_{-\infty}^{t} E_{max}^{2} \left[G_{t}^{-}(0,*) \right](\tau) d\tau + c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0,*) \right](\tau) d\tau + c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0,*) \right](\tau) d\tau$$
$$\geq c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0,*) \right](\tau) d\tau$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}[E](t) := \min_{G_{t}^{-} \in \sigma(E_{t}^{-})} W[G_{t}^{-}] = c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0,*) \right](\tau)$$

$$c \int_{-\infty}^{t} E_{max}^{2} \left[G_{t}^{-}(0,*) \right](\tau) d\tau = 0$$

$$c \int_{t}^{+\infty} E_{max}^{2} \left[G_{t}^{-}(0,*) \right](\tau) d\tau = c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0,*) \right](\tau) d\tau$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}[E](t) := \min_{G_{t}^{-} \in \sigma(E_{t}^{-})} W[G_{t}^{-}] = c \int_{t}^{+\infty} E_{max}^{2} [E_{t}^{-}(0,*)](\tau)$$

$$c \int_{-\infty}^{t} E_{max}^{2} [G_{t}^{-}(0,*)](\tau) d\tau = 0$$

$$c \int_{t}^{+\infty} E_{max}^{2} [G_{t}^{-}(0,*)](\tau) d\tau = c \int_{t}^{+\infty} E_{max}^{2} [E_{t}^{-}(0,*)](\tau) d\tau$$

$$\iff$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$F_{max}[E](t) := \min_{G_{t}^{-} \in \sigma(E_{t}^{-})} W[G_{t}^{-}] = c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0, *) \right](\tau)$$

$$c \int_{-\infty}^{t} E_{max}^{2} \left[G_{t}^{-}(0, *) \right](\tau) d\tau = 0$$

$$c \int_{t}^{+\infty} E_{max}^{2} \left[G_{t}^{-}(0, *) \right](\tau) d\tau = c \int_{t}^{+\infty} E_{max}^{2} \left[E_{t}^{-}(0, *) \right](\tau) d\tau$$

$$\Leftrightarrow$$

$$0 = E_{max} \left[G_{t}^{-}(0, *) \right](\tau) \text{ for } \tau < t$$

$$E_{max} \left[G_{t}^{-}(0, *) \right](\tau) = E_{max} \left[E_{t}^{-}(0, *) \right](\tau) \text{ for } \tau > t$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$\begin{aligned} G_{\boldsymbol{t}}^{-}\left[E_{\boldsymbol{t}}^{-}(0,*)\right] &: N_{>}\mathcal{F}\left[G_{\boldsymbol{t}}^{-}(0,*)\right] &\in \mathcal{E}_{\boldsymbol{t}}^{+}, \& \\ N_{>}\mathcal{F}\left[\left(G_{\boldsymbol{t}}^{-}-E_{\boldsymbol{t}}^{-}\right)(0,*)\right] \in \mathcal{E}_{\boldsymbol{t}}^{-}; \end{aligned}$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

$$G_{t}^{-} \left[E_{t}^{-}(0,*) \right] : N_{>} \mathcal{F} \left[G_{t}^{-}(0,*) \right] \in \mathcal{E}_{t}^{+}, \&$$
$$N_{>} \mathcal{F} \left[\left(G_{t}^{-} - E_{t}^{-} \right) (0,*) \right] \in \mathcal{E}_{t}^{-};$$
$$N_{>}(\omega) = \prod_{j=1}^{M} \frac{\left(\omega - \nu_{j}^{*} \right) \left(\omega + \nu_{j} \right)}{\left(\omega - \omega_{j} \right) \left(\omega + \omega_{j}^{*} \right)}$$

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Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$G_{t}^{-} \left[E_{t}^{-}(0,*) \right] : N_{>} \mathcal{F} \left[G_{t}^{-}(0,*) \right] \in \mathcal{E}_{t}^{+}, \&$$

$$N_{>} \mathcal{F} \left[\left(G_{t}^{-} - E_{t}^{-} \right) (0,*) \right] \in \mathcal{E}_{t}^{-};$$

$$N_{>}(\omega) = \prod_{j=1}^{M} \frac{\left(\omega - \nu_{j}^{*} \right) \left(\omega + \nu_{j} \right)}{\left(\omega - \omega_{j} \right) \left(\omega + \omega_{j}^{*} \right)} = \frac{n(\omega)}{N_{>}(-\omega)}$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

$$G_{t}^{-} \left[E_{t}^{-}(0,*) \right] : N_{>} \mathcal{F} \left[G_{t}^{-}(0,*) \right] \in \mathcal{E}_{t}^{+}, \&$$

$$N_{>} \mathcal{F} \left[\left(G_{t}^{-} - E_{t}^{-} \right) (0,*) \right] \in \mathcal{E}_{t}^{-};$$

$$N_{>}(\omega) = \prod_{j=1}^{M} \frac{\left(\omega - \nu_{j}^{*} \right) \left(\omega + \nu_{j} \right)}{\left(\omega - \omega_{j} \right) \left(\omega + \omega_{j}^{*} \right)} = \frac{n(\omega)}{N_{>}(-\omega)} \Longrightarrow$$

$$\mathcal{F} \left[G_{t}^{-}(0,*) \right] (\omega) = \sum_{j=1}^{M} \frac{-iG_{j}}{\omega + \nu_{j}} + \frac{-iG_{j}^{*}}{\omega - \nu_{j}^{*}};$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

$$\begin{aligned} G_{t}^{-}\left[E_{t}^{-}(0,*)\right] &: N_{>}\mathcal{F}\left[G_{t}^{-}(0,*)\right] &\in \mathcal{E}_{t}^{+}, \&\\ N_{>}\mathcal{F}\left[\left(G_{t}^{-}-E_{t}^{-}\right)(0,*)\right] \in \mathcal{E}_{t}^{-};\\ N_{>}(\omega) &= \prod_{j=1}^{M} \frac{\left(\omega-\nu_{j}^{*}\right)\left(\omega+\nu_{j}\right)}{\left(\omega-\omega_{j}\right)\left(\omega+\omega_{j}^{*}\right)} = \frac{n(\omega)}{N_{>}(-\omega)} \Longrightarrow\\ \mathcal{F}\left[G_{t}^{-}(0,*)\right](\omega) &= \sum_{j=1}^{M} \frac{-iG_{j}}{\omega+\nu_{j}} + \frac{-iG_{j}^{*}}{\omega-\nu_{j}^{*}};\\ \mathcal{F}\left[G_{t}^{-}(0,*)\right](\omega) &= \mathcal{F}\left[E_{t}^{-}(0,*)\right](\omega), \quad \omega = \omega_{j}, -\omega_{j}^{*}\end{aligned}$$

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Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

Summary with Addenda:

$$F_{max}\left[E\right]\left(t\right) := \min_{\substack{G_t^- \in \sigma\left(E_t^-\right)}} W[G_t^-](+\infty)$$

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Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

Summary with Addenda:

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_t^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^M G_j \exp\left(i\nu_j \left(\tau - t\right)\right) + C.C. \right) \Theta_t^{-}(\tau)$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

Summary with Addenda:

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_{t}^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^{M} G_{j} \exp\left(i\nu_{j}\left(\tau-t\right)\right) + C.C. \right) \Theta_{t}^{-}(\tau)$$

$$\sum_{j=1}^{M} \frac{-iG_j}{\omega + \nu_j} + \frac{-iG_j^*}{\omega - \nu_j^*} = \mathcal{F}\left[E_t^-(0, *)\right](\omega), \quad \omega = \omega_j, -\omega_j^*;$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

Summary with Addenda:

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

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$$\sum_{j=1}^{M} \frac{-iG_j}{\omega + \nu_j} + \frac{-iG_j^*}{\omega - \nu_j^*} = \mathcal{F}\left[E_t^-(0, *)\right](\omega), \quad \omega = \omega_j, -\omega_j^*;$$

 $n(\omega)$

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Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

Summary with Addenda:

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_{t}^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^{M} G_{j} \exp\left(i\nu_{j}\left(\tau-t\right)\right) + C.C. \right) \Theta_{t}^{-}(\tau)$$

$$\sum_{j=1}^{M} \frac{-iG_j}{\omega + \nu_j} + \frac{-iG_j^*}{\omega - \nu_j^*} = \mathcal{F}\left[E_t^-(0, *)\right](\omega), \quad \omega = \omega_j, -\omega_j^*;$$

$$n(\omega) = N_{<}(\omega)N_{<}(-\omega)$$

Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

Summary with Addenda:

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_{t}^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^{M} G_{j} \exp\left(i\nu_{j}\left(\tau-t\right)\right) + C.C. \right) \Theta_{t}^{-}(\tau)$$

$$\sum_{j=1}^{M} \frac{-iG_j}{\omega + \nu_j} + \frac{-iG_j^*}{\omega - \nu_j^*} = \mathcal{F}\left[E_t^-(0, *)\right](\omega), \qquad \omega = \omega_j, -\omega_j^*;$$
$$\omega(\omega) = N_{<}(\omega)N_{<}(-\omega) = \prod_{j=1}^{M} \frac{\left(\omega - \nu_j^*\right)\left(\omega + \nu_j\right)}{\left(\omega - \nu_j^*\right)\left(\omega - \nu_j^*\right)} \times C.C.$$

$$\prod_{j=1}^{k} \left(\omega - \omega_j \right) \left(\omega + \omega_j^* \right) \quad \text{for all } i \in \mathbb{N}$$

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Optimal Energy Injection Pulses: Steering the Maximum Free Energy in Bulk

Designer Pulse for Optimal Energy Injection: Semi-Infinite Slab $z \in [0, +\infty)$

Summary with Addenda:

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_{\mathbf{t}}^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^{M} G_{j} \exp\left(i\nu_{j} \left(\tau - \mathbf{t}\right)\right) + C.C. \right) \Theta_{\mathbf{t}}^{-}(\tau)$$

$$\sum_{j=1}^{M} \frac{-iG_j}{\omega + \nu_j} + \frac{-iG_j^*}{\omega - \nu_j^*} = \mathcal{F}\left[E_t^-(0, *)\right](\omega), \quad \omega = \omega_j, -\omega_j^*;$$

$$h(\omega) = N_{<}(\omega)N_{<}(-\omega) = \prod_{j=1}^{M} \frac{\left(\omega - \nu_j^*\right)\left(\omega + \nu_j\right)}{\left(\omega - \omega_j\right)\left(\omega + \omega_j^*\right)} \times C.C. = \operatorname{Re}\sqrt{\varepsilon(\omega)}$$

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$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_{t}^{\checkmark}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^{M} \check{G}_{j} \exp\left(i \nu_{j} (\tau - t)\right) + C.C. \right) \Theta_{t}^{\neg}(\tau)$$

Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_t^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^M \check{G}_j \exp\left(i \check{\nu}_j (\tau - t)\right) + C.C. \right) \Theta_t^{-}(\tau)$$

$$\sum_{j=1}^{M} \frac{-i \check{G}_{j}}{\omega + \nu_{j}} + \frac{-i \check{G}_{j}^{*}}{\omega - \nu_{j}^{*}} = \mathcal{F} \left[E_{t}^{-}(0, *) \right] (\omega), \qquad \omega = \check{\omega}_{j}, -\check{\omega}_{j}^{*};$$

Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

Designer Media for Optimal Energy Injection by a Specified Pulse: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_t^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^M \check{G}_j \exp\left(i \check{\nu}_j (\tau - t)\right) + C.C. \right) \Theta_t^{-}(\tau)$$

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$$\sum_{j=1}^{M} \frac{-i \check{G}_{j}}{\omega + \nu_{j}} + \frac{-i \check{G}_{j}^{*}}{\omega - \nu_{j}^{*}} = \mathcal{F} \left[E_{t}^{-}(0, *) \right] (\omega), \qquad \omega = \check{\omega}_{j}, -\check{\omega}_{j}^{*};$$

$$\operatorname{Re}\sqrt{\varepsilon(\omega)}$$

Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

Designer Media for Optimal Energy Injection by a Specified Pulse: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_t^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^M \check{G}_j \exp\left(i \nu_j (\tau - t)\right) + C.C. \right) \Theta_t^{-}(\tau)$$

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$$\sum_{j=1}^{M} \frac{-i \check{G}_{j}}{\omega + \nu_{j}} + \frac{-i \check{G}_{j}^{*}}{\omega - \nu_{j}^{*}} = \mathcal{F} \left[E_{t}^{-}(0, *) \right] (\omega), \qquad \omega = \check{\omega}_{j}, -\check{\omega}_{j}^{*};$$

$$\operatorname{Re}\sqrt{\varepsilon(\omega)} = n(\omega)$$

Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

Designer Media for Optimal Energy Injection by a Specified Pulse: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_t^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^M \check{G}_j \exp\left(i \check{\nu}_j (\tau - t)\right) + C.C. \right) \Theta_t^{-}(\tau)$$

$$\sum_{j=1}^{M} \frac{-i \check{G}_{j}}{\omega + \nu_{j}} + \frac{-i \check{G}_{j}^{*}}{\omega - \nu_{j}^{*}} = \mathcal{F} \left[E_{t}^{-}(0, *) \right](\omega), \qquad \omega = \check{\omega}_{j}, -\check{\omega}_{j}^{*};$$

$$\operatorname{Re}\sqrt{\varepsilon(\omega)} = n(\omega) = \left| \prod_{j=1}^{M} \frac{\left(\omega - \nu_{j}^{*}\right)\left(\omega + \nu_{j}\right)}{\left(\omega - \omega_{j}\right)\left(\omega + \omega_{j}^{*}\right)} \right|$$

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Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

Designer Media for Optimal Energy Injection by a Specified Pulse: Semi-Infinite Slab $z \in [0, +\infty)$

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_{t}^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^{M} \check{G}_{j} \exp\left(i \nu_{j}(\tau-t)\right) + C.C. \right) \Theta_{t}^{-}(\tau)$$

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$$\sum_{j=1}^{M} \frac{-i \check{G}_{j}}{\omega + \nu_{j}} + \frac{-i \check{G}_{j}^{*}}{\omega - \nu_{j}^{*}} = \mathcal{F} \left[E_{t}^{-}(0, *) \right](\omega), \qquad \omega = \check{\omega}_{j}, -\check{\omega}_{j}^{*};$$

$$\frac{1}{2}\left(\sqrt{\varepsilon(\omega)} + \sqrt{\varepsilon(-\omega)}\right) = n(\omega)$$

Steering the Maximum Free Energy in Bulk by a Specified Pulse: What is the Medium?

$$F_{max}[E](t) := \min_{G_t^- \in \sigma(E_t^-)} W[G_t^-](+\infty)$$

$$G_t^{-}(0,\tau) = \sqrt{2\pi} \left(\sum_{j=1}^M \check{G}_j \exp\left(i \check{\nu}_j (\tau - t)\right) + C.C. \right) \Theta_t^{-}(\tau)$$

$$\sum_{j=1}^{M} \frac{-i \check{G}_{j}}{\omega + \nu_{j}} + \frac{-i \check{G}_{j}^{*}}{\omega - \nu_{j}^{*}} = \mathcal{F} \left[E_{t}^{-}(0, *) \right](\omega), \qquad \omega = \check{\omega}_{j}, -\check{\omega}_{j}^{*};$$

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Complications for Optimal Energy Injection Designs: Finite Slab $z \in [0, L]$

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$$N_{>}(\omega)N_{>}(-\omega) = n_{[0,L]}(\omega)$$

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Complications for Optimal Energy Injection Designs: Finite Slab $z \in [0, L]$

$$\begin{split} N_{>}(\omega)N_{>}(-\omega) &= n_{[0,L]}(\omega) \\ N_{>}(\omega) &\in \mathcal{E}_{0}^{+}, 1/N_{>}(\omega) \in \mathcal{E}_{0}^{-} \end{split}$$

Complications for Optimal Energy Injection Designs: Finite Slab $z \in [0, L]$

$$N_{>}(\omega)N_{>}(-\omega) = n_{[0,L]}(\omega) = \frac{\omega \operatorname{Im}\left[\chi(\omega)\right]}{c} \int_{0}^{L} |r(\omega, z)|^{2} dz$$
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$$r(\omega,z) = \frac{\widehat{E}(z,\omega)}{\widehat{E}(0,\omega)} = \frac{(\varepsilon^{\frac{1}{2}}+1)\exp\left(\frac{-i\omega}{c}\varepsilon^{\frac{1}{2}}z\right) + (\varepsilon^{\frac{1}{2}}-1)\exp\left(\frac{+i\omega}{c}\varepsilon^{\frac{1}{2}}z\right)}{(\varepsilon^{\frac{1}{2}}+1)\exp\left(\frac{-i\omega}{c}\varepsilon^{\frac{1}{2}}L\right) + (\varepsilon^{\frac{1}{2}}-1)\exp\left(\frac{+i\omega}{c}\varepsilon^{\frac{1}{2}}L\right)}$$

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$$\longleftrightarrow \text{ Global "Constitutive" Relation/Propagation:}$$

$$\begin{split} N_{>}(\omega)N_{>}(-\omega) &= n_{[0,L]}(\omega) = \frac{\omega \mathrm{Im}\left[\chi(\omega)\right]}{c} \int_{0}^{L} |r(\omega,z)|^{2} dz \\ N_{>}(\omega) &\in \mathcal{E}_{0}^{+}, 1/N_{>}(\omega) \in \mathcal{E}_{0}^{-} \\ r(\omega,z) &= \frac{\widehat{E}(z,\omega)}{\widehat{E}(0,\omega)} = \frac{(\varepsilon^{\frac{1}{2}}+1)\exp\left(\frac{-i\omega}{c}\varepsilon^{\frac{1}{2}}z\right) + (\varepsilon^{\frac{1}{2}}-1)\exp\left(\frac{+i\omega}{c}\varepsilon^{\frac{1}{2}}z\right)}{(\varepsilon^{\frac{1}{2}}+1)\exp\left(\frac{-i\omega}{c}\varepsilon^{\frac{1}{2}}L\right) + (\varepsilon^{\frac{1}{2}}-1)\exp\left(\frac{+i\omega}{c}\varepsilon^{\frac{1}{2}}L\right)} \\ &\longleftarrow \text{Global "Constitutive" Relation/Propagation:} \\ \widehat{E}''(z,\omega) &= \frac{-\omega^{2}}{c^{2}}\left\{1 + \chi(\omega)\left[\Theta^{+}(z) - \Theta^{+}(z-L)\right]\right\}\widehat{E}(z,\omega); \end{split}$$

Complications for Optimal Energy Injection Designs: Finite Slab $z \in [0, L]$

$$\begin{split} N_{>}(\omega)N_{>}(-\omega) &= n_{[0,L]}(\omega) = \frac{\omega \mathrm{Im}\left[\chi(\omega)\right]}{c} \int_{0}^{L} |r(\omega,z)|^{2} dz \\ N_{>}(\omega) &\in \mathcal{E}_{0}^{+}, 1/N_{>}(\omega) \in \mathcal{E}_{0}^{-} \\ r(\omega,z) &= \frac{\widehat{E}(z,\omega)}{\widehat{E}(0,\omega)} = \frac{(\varepsilon^{\frac{1}{2}}+1)\exp\left(\frac{-i\omega}{c}\varepsilon^{\frac{1}{2}}z\right) + (\varepsilon^{\frac{1}{2}}-1)\exp\left(\frac{+i\omega}{c}\varepsilon^{\frac{1}{2}}z\right)}{(\varepsilon^{\frac{1}{2}}+1)\exp\left(\frac{-i\omega}{c}\varepsilon^{\frac{1}{2}}L\right) + (\varepsilon^{\frac{1}{2}}-1)\exp\left(\frac{+i\omega}{c}\varepsilon^{\frac{1}{2}}L\right)} \\ &\longleftarrow \text{ Global "Constitutive" Relation/Propagation:} \\ &\widehat{E}''(z,\omega) = \frac{-\omega^{2}}{c^{2}}\left\{1 + \chi(\omega)\left[\Theta^{+}(z) - \Theta^{+}(z-L)\right]\right\}\widehat{E}(z,\omega); \\ &\widehat{E}(0,\omega) \text{ prescribed.} \end{split}$$

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Summary

 Min Free Energy is maximum available for return to the field after some specified time

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 Designing pulses and media for Max Extraction Energy involves only Kramers-Kronig-like calculus

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- Designing pulses and media for Min Injection Energy is algorithmic for the semi-infinite slab

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Designing media for Min Injection Energy is "hard" for the finite slab

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- Future Prospects
 - Demonstrate the Max-Min Free Energy Carnot Cycle: Cycle gives optimally long-lived/slow pulse in the medium
 - Develop directional Min Free Energy, etc.: Max extractable energy leaving right-hand/output side of finite slab, etc. Directional Carnot Cycle gives slowest right-going pulse.

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 Broadband Pulse

Broadband pulse encompassing two nearby absorption resonances



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Outline Thermodynamics of Dispersion Local Free Energies Global Free Energies Designer Pulses Designer Media Summary I 000 00 00 00 00 00 Narrow-band Pulse

Two absorption resonances encompassing narrow-band



 $E(t) = E_0 e^{-t^2/T^2} \cos(\bar{\omega}t)$ $\chi(\omega) = \sum_{n=1}^{2} \frac{f_n \omega_{p_n}^2}{\omega_n^2 - i\gamma_n \omega - \omega^2}$ $\gamma_1 = \gamma_2 = \gamma$; $\omega_1 = 40\gamma$ $\omega_2 = 60\gamma$: $\bar{\omega} = 50\gamma$ $T = 1/\gamma$. $f_1 \omega_{p_1}^2 = f_2 \omega_{p_2}^2 = 100 \gamma^2$ $u_{\text{int}}(t) = W[E](t)$ $u_{\rm irrec}(t) = Q_{\rm min}[E](t)$ $u_{\text{waste}}(t) = Q_{\max}[E](t)$

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Broad-band pulse encompassing mixed passive/active resonances



$$E(t) = E_0 e^{-t^2/T^2} \cos(\bar{\omega}t)$$

$$\chi(\omega) = \sum_{n=1}^2 \frac{f_n \omega_{p_n}^2}{\omega_n^2 - i\gamma_n \omega - \omega^2}$$

$$\gamma_2 = 0.1\gamma_1; \ \omega_1 = \omega_2 = 10\gamma_1$$

$$\bar{\omega} = 10\gamma_1$$

$$T = 2.5/\gamma_1.$$

$$f_1 \omega_{p_1}^2 = 10\gamma_1^2; \ f_2 \omega_{p_2}^2 = -.99\gamma_1^2$$

$$u_{\text{int}}(t) = W[E](t)$$

$$u_{\text{irrec}}(t) = Q_{\text{min}}[E](t)$$

$$u_{\text{waste}}(t) = Q_{\text{max}}[E](t)$$

Outline Thermodynamics of Dispersion Local Free Energies Global Free Energies Designer Pulses Designer Media Summary L 000 00 00 00 00 00 Broad-band Pulse near EIT

Broad-band pulse encompassing mixed passive/active



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