## ELEMENTARY LINEAR ALGEBRA MATH 313 SECTION 001 SPRING 2010<sup>1</sup>

<u>Introduction to Linear Algebra.</u><sup>2</sup> "Linear algebra is the mathematical treatment of linear equations in any number of variables. Virtually any industrial or scientific problem can be modeled in terms of these equations. Suppose for instance we have variables x(1), ..., x(6) and these are subject to two equations.

1. 
$$x(1) + 2x(2) - x(3) + 4x(4) + x(5) - 8x(6) = 0$$

2. 
$$3x(1) - x(2) + x(3) + 7x(4) - x(5) + 6x(6) = 0$$

We can think geometrically. If we had only three variables x(1), x(2), x(3) we could think of (x(1), x(2), x(3)) as a point in 3-dimensional space (a vector) and we have an apparatus of length, distance, angle, straight lines, planes, and so on. Similarly we think of (x(1),...,x(6)) as a point in six-dimensional space and we develop an apparatus that includes all the above ideas in a more general form: length, distance, inner product, subspace. You will learn to think of 1 and 2 as representing a subspace whose dimension is 4 and write down a parametric solution in terms of a basis. You will also think of 1 and 2 as the null space of a matrix and as the null space of a linear mapping. The ideas that you learn will be illustrated by going into two or three dimensions."

Class Meetings: Section 001—MWF 2-3:50 pm, 2113 JKB

**Instructor:** Dr. Glasgow, Office 344 TMCB<sup>3</sup>, Tel 422-9086

**Text:** Howard Anton, Elementary Linear Algebra, 9<sup>th</sup> ed., John Wiley &

Sons, 2005

**Office Hours:** 12:00pm - 1:00 pm, 4:00pm—6:00 pm MWF<sup>4</sup> **Research Hours:** All day Tuesday and Thursday (and Saturday)<sup>5</sup>

**Exams.** There will be three midterm exams, taken in the testing center, with dates indicated on the attached calendar. Each of these exams will be worth 15% of your grade, for a total of 45%. The midterms are not timed, and may take at least 3 hours each. No calculators will used on these assessments, or on the final exam. (Computational

<sup>2</sup> A description by Roger Baker.

<sup>&</sup>lt;sup>1</sup> "The year we make contact."

<sup>&</sup>lt;sup>3</sup> For now this is the office, but the Math Department is making a no-fixed-point permutation of offices this term so expect a change.

<sup>&</sup>lt;sup>4</sup> These will be shared with students taking differential equations.

<sup>&</sup>lt;sup>5</sup> All mathematics faculty have been asked to communicate to their students the times during which they have "no-contact" activities. The idea is to remind students that they attend a major University at which faculty are expected to perform many functions, including performing research. Please make sacrifices to attend the limited office hours. I am more than happy to help when I can.

problems will be engineered to have solution components that are integers as often as possible, and on other, theoretical problems calculators will be unhelpful.)

**Final Exam.** The final exam will be 2 hours and held in class, the date for your section indicated on the attached calendar. It will be more cumulative in nature than the midterms and worth 25% of your grade.

**Quizzes, Reading and Lectures.** Daily reading quizzes will be worth 10% of your grade. They will be available on BYU Blackboard and will have deadlines insuring that you *read before class* the material to be lectured on. The section to be lectured on for any given date is indicated in the calendar included below.

**Homework.** Homework is due according to the attached calendar. The homework should be turned in at the beginning of class on the day indicated on the calendar. *Late homework will not be accepted.* Homework will be worth 20% of your grade.

**Self-assessment.** Scores for each of the above will be posted on BYU Blackboard. These will be easily interpreted without sophisticated calculation because we weight each of the types of scores equally—a total of 450 points possible for midterms (150 points each), 250 for the final, 200 for homework, and 100 for quizzes. In order to accommodate illness or other unforeseen circumstances, each of the 35 homework assignments (in a normal semester—around half that in a term) will be worth 6 points (in a normal semester—around twice that in a term), the sum of these being 210, i.e. 10 points more than the 200 required for a "perfect score". So there is the possibility for extra-credit, since all of your score, even that fraction over 200, will be counted in the 1000 point total. A similar description of quizzes holds: each of the approximately 40 quizzes (in a normal semester) will be worth 3 points (in a normal semester), providing some buffer for illness and the like. PLEASE DO NOT ASK FOR OTHER LENIENCIES.

**Grade Distribution.** The following percentages guarantee the indicated grade:

93% A, 90% A-, 87% B+, 83% B, 80% B-, 77% C+, 73% C, 70% C-, 67 % D+, 63 % D, 60 % D-. These are 930, 900, 870, 830, 800, 770, 730, 700, 670,630, and 600 points, respectively. Thus there is no "Darwinism" in this course, and you should feel comfortable helping your classmates understand the material, and otherwise striving to create a team spirit.

General Considerations for non-math majors: This course should be viewed similar to the way one typically views a course in English, Social Science, History, or any other course where synthesis of complex ideas into readable prose is routinely expected. The fact that our general discipline (Mathematics) and specific subject (Linear Algebra) are vastly less ambiguous and subjective than many others only makes the need for precise, convincing communication even more important. While some of this communication should be in the form of specialized mathematical symbols seen in the usual "symbol

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<sup>&</sup>lt;sup>6</sup> Math majors usually already understand the point of this paragraph. The relevant idea has been communicated to them as "mathematical maturity" in courses such as 290.

pushing," a large fraction of it should be in the standard form generally used by the average educated person—English sentences with all of their essential/defining components. Before you turn in your homework, you should be able to read it out loud and confirm that a person who can only hear your voice, but who is otherwise attentive, can understand your development. In this way you can assure that everything you write, with or without specialized symbols, is part of a complete idea—a complete sentence. Can you explain why the following two lines do not constitute a complete sentence/idea

$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

yet the next two do?

$$ax^{2} + bx + c = 0 \implies$$

$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}.$$

The symbol "\Rightarrow" means "implies that." Writing out "implies that" is even better for our purposes. Which of these two strings of symbols corresponds more closely to what a mathematician (or engineer, physicist, economist, etc) might write in a communication to colleagues? Again, an English sentence would be even better.

Ensuring each item written on homework or exam is part of a complete sentence will allow you to find that mathematics can be approached quite "cerebrally," as much as any other discipline, i.e. without reliance on rote skill. In some circumstances students will find they finally understand mathematics. On a more mundane note, your homework and test scores will be reduced if not approached in this "complete sentence" format. It is a requirement of this course that you *think* and, so, that you *write*.

**Calculators** can be used to check your Homework. (But they will not be helpful for the more important, theoretical/structural problems.) However, work has to be shown in quizzes and homework to get any credit. "Showing work" means, among other things, constructing complete ideas/sentences.

### **Course Learning Objectives**

The core contents of this course that should be learned all sections and/or are required for subsequent courses in the program—i.e. things you need to understand and remember—are

- Systems of linear equations and Gaussian elimination
- Matrix arithmetic
- Linear transformations

- Invertible matrices and transformations
- Determinants
- Vector spaces and Subspaces
- Linear independence, bases, dimension, and rank
- Inner products, orthogonal sets, and least squares
- Eigenvalues, eigenvectors, and the characteristic equation of a square matrix
- Diagonalization and the spectral theorem for symmetric matrices

The specific outcomes/skills/competencies that you should be able to demonstrate at the completion of the course (i.e. what you should be able to apply, analyze, evaluate, or create) are

- Articulate basic definitions and theorems
- Solve systems of linear equations via Gaussian elimination
- Find the standard matrix of a linear transformation that is described geometrically
- Find the inverse of a matrix using either row operations or cofactors
- Find the determinant of a matrix using row operations and/or cofactor expansion
- Determine whether or not a given set is a subspace
- Determine whether a given set of vectors is linearly independent, spans a specified subspace, is a basis, etc.
- Perform computations with inner products
- Find the eigenvalues and eigenvectors of 2 x 2 or 3 x 3 matrices
- Determine whether or not a matrix is diagonalizable, and if so, find a matrix that diagonalizes it. Find a matrix that orthogonally diagonalizes a symmetric matrix
- Prove or disprove a given statement

## Semester Lecture, HW, Exam and General Academic Schedule. (Next Page)

April 2010										
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday				
25	26	27	28 <b>1.1,1.2</b>	29	30 <b>1.3,1.4</b> 1.1:1,4,5,7,8, 11 1.2:1,2,4,8,22, 25					

## May 2010

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
						1
2	3 <b>1.5, 1.6</b> 1.3: 1,2,a-c on 3- 6,12,13,18,20 1.4: 4,5,6,11,13- 17,21,31	4 Add/Drop Deadline	5 <b>1.7, 2.4</b> 1.5: 1-4,8,16, 22 1.6:2,3,6,9,11, 23,29	6	7 <b>2.1,2.2</b> 1.7: 1,3-5, 8, 15,18,22, 30 2.4: 1, 2, 3, 7, 9, 12, 13, 21, 23	8
9	10 <b>2.3, 4.1</b> 2.1: 1, 5, 7, 10, 11,14, 25, 26, 34,35 2.2: 2, 3, 4, 5, 6, 11, 12, 13, 19, 20	11	12 <b>Review</b> 2.3: 3,5,7,12,16, 18,20,22,23 4.1: 2, 4, 7-9, 14, 19, 37	13 <b>Test I</b> :1.1-2.2	14 <b>4.2,4.3</b> Test I:1.1-2.2	15
16	17 <b>5.1, 5.2</b> 4.2: 1, 3, 5, 6, 12, 13, 19, 23 4.3: 1-4, 6, 7, 13, 16, 25	18	19 <b>5.3, 5.4</b> 5.1: 1, 5, 7-10, 12, 15, 28, 29 5.2: 1-14,19, 20, 22, 23	20	21 <b>5.5</b> , <b>5.6</b> 5.3: 1-4, 7, 9- 12, 14-16, 18,21, 24 5.4: 3-5, 7, 9- 14, 18,19, 21, 23,24	22
23	24 <b>6.1, 6.2</b> 5.5: 1-3, 6, 7, 11 5.6: 2-7, 9, 13,17	25	26 <b>Review</b> 6.1: 3, 4, 9, 15, 17,23,27 6.2: 1, 5, 8, 9, 10, 18, 19, 21, 24	27 Test II:2.3-5.6	28 <b>6.3</b> , <b>6.4</b> Test II:2.3-5.4	29
30	31Holiday					

#### June 2010 Sunday Monday Tuesday Wednesday Thursday Friday Saturday 5 1 Withdraw 2 6.5,6.6 3 4 7.1, 7.2, 7.3 Deadline 6.3: 4, 5(a), 6.5: 1,2,3,6,8, 9,10,11,16, 14 17, 18, 26, 29 6.6: 3, 4, 13, 6.4:2,3,5,14, 14 7 Review 11 8.4,8.5 9 8.1, 8.2, 8.3 10 12 8.1: 3, 5, 6, 7, 7.1: 1-9(a), 10-Discontinuance 12, 19, 20, 24, 16, 19, 30, 31 Deadline on Test III:6.1-Test III:6.1-25 8.2: 5, 10, 11, Monday 6.6 6.6 7.2: 1-3, 5, 14, 12, 17, 22, 15, 21, 25 23, 25 8.3: 2, 4, 9, 7.3: 1(a,b), 2, 4, 6, 10, 11, 13, 16, 19, 24 14, 15 14 15 Exam 18 13 17 Final 19 16 8.4: 1a, 2a, 3a, Preparation Exam 9, 14, 16 Day 3-5pm 8.5:13,14 20 21 22 23 24 25 26

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# Studying Mathematics at the University Dr. Lynn E. Garner, Former Chair, Department of Mathematics, Brigham Young University

Brigham Young said, "Education is the power to think clearly, the power to act well in the world's work, and the power to appreciate life." Mathematics and quantitative reasoning are fundamental to these three powers, especially in our technological world in which reality is described in increasingly mathematical terms.

The goals of university mathematics courses are not only to develop manipulative skills in arithmetic, algebra, etc., but also to impart an understanding of mathematical ideas in new contexts and with much more flexibility. For example, most of you expect to use mathematics as a fundamental tool. The power to use mathematics effectively in your discipline requires you to have

- a conceptual understanding of both mathematical principles and the principles of your discipline,
- the ability to translate features of your discipline into a mathematical model,
- the knowledge and skill to apply mathematics to the model, and
- the ability to express the mathematical results as predictions in the discipline.

As you see, manipulative skills are necessary but inadequate without conceptual understanding, and this is true in any major. If you wish to study mathematics itself, the expectation is that you will not only master the knowledge and skills of the mathematics courses, but also learn to communicate in mathematical terms. The language and theory of proof will become critically important to you.

Attitudes toward learning in mathematics courses must be consistent with these goals. In high school, most learning took place in the classroom and students usually didn't spend as much time on homework as in class. One who was attentive in class could usually succeed with modest effort. At the university, most of your learning will take place *outside* the classroom and you will be expected to spend at least *twice* as much time on homework and reading as you spend in class. In addition to being attentive in class, you will have to exert considerable effort outside of class in order to succeed. You will be expected to learn the basic ideas in a course from the textbook because there is typically not enough time to cover all of them in class. And, given this change in the location of learning activities, it is obvious that your instructor is no longer primarily responsible for what you learn; *you* are. Finally, go beyond solving problems like the examples in your text. The problems you will meet on the job have not yet been solved and are not in the textbooks. If all you can do is solve text problems, you will be replaced by a computer. Practice solving problems you have not seen before. Learn to think; that, a computer cannot do.

Taking responsibility for your own learning includes gathering pertinent information, enhancing the learning environment, being committed to academic integrity, and using responsibly the exceptions afforded by extenuating circumstances.

You are responsible to know all the requirements in your major, your minor, the
university generally, and every course you take. Verify advice from anyone else
with authoritative sources: your instructor, the syllabus, the textbook, or your

- advisement center. Ignorance is never an excuse. Exert every effort to master the material of each course. Strive for excellence. You can study harder than you now know.
- Your actions should enhance the learning environment. Avoid distracting activities in the classroom, the library, and the dormitory. Always prepare for class by completing homework and reading assignments.
- Academic integrity means that you will not allow yourself or others to profit from information to which you or they have no right. Not only do you avoid plagiarism, but you do not receive or give inappropriate information about tests, quizzes, or homework. Grades are given only on the basis of academic performance; to ask for grades on any other basis is a form of academic dishonesty.
- Extenuating circumstances include serious illness, family emergency, and official university business. Instructors usually allow you to make up work missed because of extenuating circumstances, but do not expect heroic efforts in your behalf; some activities can't be made up. Arrange ahead of time or as soon as possible afterward. The timing of an extenuating circumstance may be critical, so act quickly.

## Strategies for learning in mathematics courses include

- managing your time, now your most precious resource, by observing and adjusting your use of it;
- making sure you are in the appropriate class and have the proper prerequisites;
- studying with classmates, teaching each other the principles involved and discussing difficult concepts;
- getting help after reasonable effort, without wasting too much time "spinning your wheels;"
- being willing to review on your own time topics you have seen but have forgotten;
- using instructor office hours and the Math Lab effectively;
- reading the textbook for basic ideas, additional information, and more examples;
   and
- Making efficient and responsible use of the library and technology.