Math 303 : Engineering Math 2

CHAPTER 1

Math Models. Direction Fields.

Math models are usually based on principles or laws of the natural world.

relationships involving rates of change

In math form: Equations involving derivatives → Differential Equations describing Physical process.

Examples:

a) Sound waves

b) Conduction of heat.

c) Fluid motion

d) Population dynamic

e) Elastic waves.

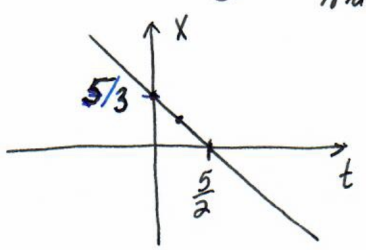
Algebraic Equations

a) $3x + 2 = 5$

$x = 1$ Soln. (only one)

b) $3x + 2t = 5$

$\Rightarrow x = \dfrac{5-2t}{3}$ Infinitely many Solns.



c) $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3, \quad x = 2$

Two Solutions.

d) $\begin{cases} 2x - 5y = 4 \\ 7x + 2y = 3 \end{cases}$

Differential Equations.

a) $3x'(t) + 2 = 5$

$x(t) = t$ Soln.

$x(t) = t + c$ Soln. for any c. infinitely many Solns.

b) $3x'(t) + 2t = 5$

$x'(t) = \dfrac{5-2t}{3}$

$x(t) = \dfrac{5}{3}t - \dfrac{\cancel{2}}{3}\dfrac{t^2}{\cancel{2}} + C$

So $\boxed{x(t) = \dfrac{5}{3}t - \dfrac{t^2}{3} + C}$ infinitely many solns.

c) $x''(t) - 5x'(t) + 6x(t) = 0$

$x_1(t) = e^{3t}$ CHAP 3

$x_2(t) = e^{2t}$

$\boxed{x(t) = C_1 e^{3t} + C_2 e^{2t}}$ Infinitely many Solns.

d) $\begin{cases} x'(t) = 2x - 5y \\ y'(t) = 7x + 2y \end{cases}$?

CHAP 7.

Example 1. A Falling Object. Mathematical Model

We want to obtain a mathematical equation describing the motion (we want to know the velocity v at any instance of time t)

v_0: Initial Velocity

t: time.

$v(t)$: Velocity at time t.

Choose a unit System

$[t]$ = Seconds or s.

$[v] = \dfrac{\text{meters}}{\text{Second}}$ or $\dfrac{m}{s}$

Assume $v(t) > 0$ (why?)

- we want to know $v(t)$.

What do we usually know?

a) weight (force) $= mg$, b) Gravity acceleration $= g$

c) Air resistance or drag force (force) $= \gamma\, v(t)$

where γ: drag Coefficient.

γ depends on the object geometry. It is a known constant value

What else do we know that may lead to a differential equation?

Physical law: Newton's second law.

$$\sum \text{forces} = \text{mass} \times \text{acceleration}$$

$\downarrow$ Algebraic Sum

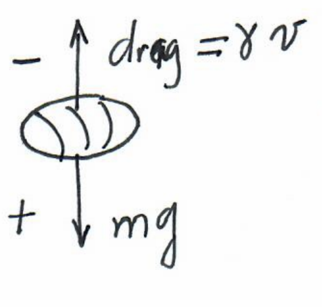
or $$\sum F = ma$$

If we use MKS unit system: $[m] = kg$, $[v] = \frac{m}{s}$

$[F] = kg \cdot \frac{m}{s^2}$ $\qquad [a] = \frac{m}{s^2}$

Now, $a = \frac{dv}{dt}$ (rate of change of velocity with respect to time).

Force diagram:



Newton:

$\rightarrow$ $$ma = mg - \gamma v$$

or $$\boxed{m \frac{dv}{dt}(t) = mg - \gamma v(t)} \quad (1)$$

Equ. (1) is a DE

g is experimentally determined: $g \simeq 9.8 \frac{m}{s^2}$ (near earth surface)

Air resistance units: $[\gamma \vec{v}] = Kg \cdot \frac{m}{s^2}$ (force) $\Rightarrow [\gamma] = \frac{Kg}{s}$

γ: depends on object (constant)

mass, "m", can be determine if the weight "mg" is known.

Thus, only unknown is $v(t)$.

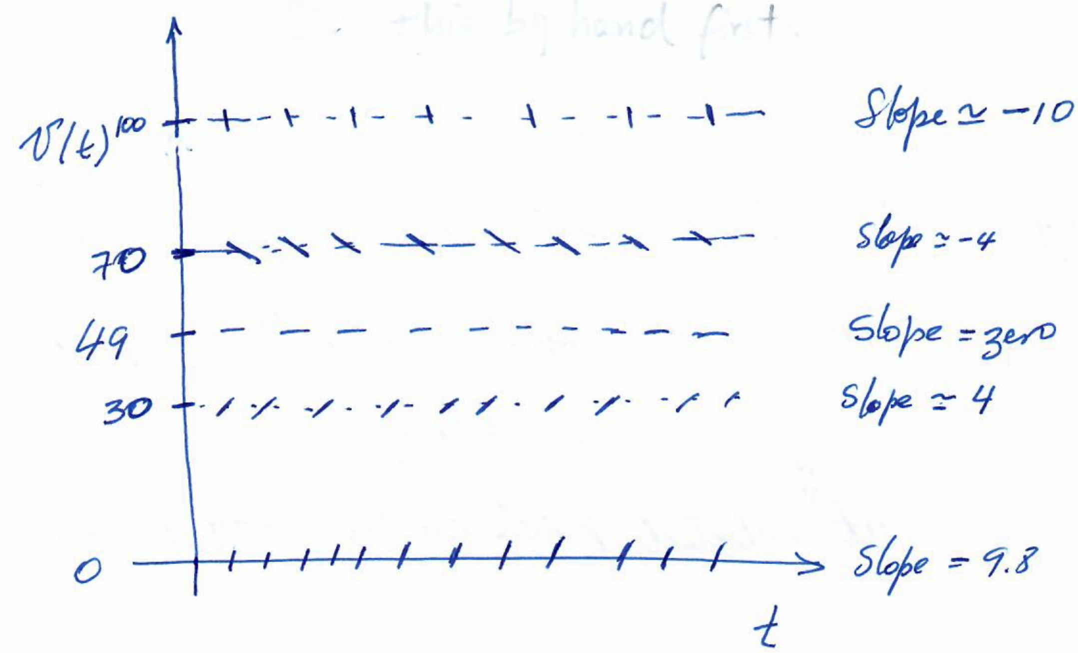
<u>Assume</u> $\boxed{mg = 98 \text{ Newt}} \Rightarrow \boxed{m = 10\,Kg; \text{ and } \gamma = 2\,\frac{Kg}{s}}$

Therefore, (1) reduces to

$$10 \frac{dv}{dt}(t) = 98 - 2v(t) \Rightarrow \boxed{\frac{dv}{dt}(t) = 9.8 - \frac{1}{5}v(t)}$$

$$\Rightarrow \boxed{\frac{dv}{dt}(t) = \frac{1}{5}\left(49 - v(t)\right)} \qquad (2)$$

<u>Concept of direction field.</u>



Direction FIELD.

$v'(t)$ is slope of tang. line to the graph of $v(t)$ at time t.

Show Example in MAPLE Worksheet.

It suggests there is a constant 'solution' to (2) close to $v \simeq 50$

In fact, if $v(t) \equiv 49$

$\Rightarrow \dfrac{dv}{dt}(t) = 0$ and also rhs $= \dfrac{1}{5}\left(49 - v(t)\right) = 0.$

$0 = 0$ identity $\Rightarrow$ $v(t) \equiv 49$ is a soln. of (2).

Since, $v(t) \equiv 49$ doesn't change with time is called an "equilibrium solution".

Definition of Direction Field.

A set of short line segments starting at selected points (t, y) of the t-y plane with the same slope as the tangent line to the graph of the solution of the ODE:

$$y'(t) = f\left(t, y(t)\right).$$

Many other nonconstant solutions could be drawn from the direction field:

Start with an "initial velocity"

$v(0) = v_0$ in the y-axis

and draw a curve tangent to the direction field as t grows.