1.2 Solutions of Some DE. General Soln. Integral Curves Initial Conditions (IC) and initial value problems (IVP)

$$v'(t) = \frac{dv}{dt}(t) = \frac{1}{5}\left[49 - v(t)\right] = -\frac{1}{5}\left[v(t) - 49\right] \quad (1)$$

If $v(t) \neq 49$

then, $\frac{1}{v(t)-49} v'(t) = -\frac{1}{5}$

$\int dt$: $\int \frac{v'(t)}{v(t)-49}\, dt = \int -\frac{1}{5}\, dt$

then, $\ln|v(t)-49| = -\frac{1}{5}t + c$

$e^{\bigcirc}$: $|v(t)-49| = e^{-\frac{1}{5}t+c} = e^{-t/5} e^{c} = c_1 e^{-t/5}$

then, $v(t) - 49 = \pm c_1 e^{-t/5} = C_2 e^{-t/5}$

thus, $\boxed{v(t) = C_2 e^{-t/5} + 49}$ (2)

<u>Thm.-</u>

$$f(x)>0$$

$$\left[\ln(f(x))\right]' = \frac{1}{f(x)} f'(x).$$

<u>Corol.-</u> If $f(x)\neq 0$

$$\left[\ln|f(x)|\right]' = \begin{cases} \left[\ln(f(x))\right]' = \dfrac{f'(x)}{f(x)}, & f(x)>0 \\ \left[\ln(-f(x))\right]' = \dfrac{1}{-f(x)}(-f'(x)), & f(x)<0 \end{cases}$$

$$= \frac{f'(x)}{f(x)} \quad \text{in the two cases.}$$

Therefore,

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|, \qquad f(x)\neq 0.}$$

In particular,

$$\int \left[\frac{1}{v(t)-49}\,\frac{dv}{dt}\right]dt = \ln|v(t)-49|$$

and

$$\int \left[\frac{1}{49-v(t)}\,\frac{dv}{dt}\right]dt =$$

$$-\int \left(\frac{1}{v(t)-49}\,\frac{dv}{dt}\right)dt = -\ln|v(t)-49|$$

$$\boxed{v'(t) = \frac{1}{5}\left[49 - v(t)\right]} \qquad (1)$$

Solns: $\boxed{v(t) = C_2 e^{-t/5} + 49}$, (2') if $v(t) \not\equiv 49$

(infinite many solutions)

Verification:

$$Lhs = v'(t) = -\frac{C_2}{5} e^{-t/5} \qquad (1)$$

$$\frac{1}{5}\left[49 - v(t)\right] = \frac{1}{5}\left[\cancel{49} - C_2 e^{-t/5} + \cancel{49}\right]$$

$$= -\frac{C_2}{5} e^{-t/5}. \qquad (2)$$

Thus,

$$Lhs = -\frac{C_2}{5} e^{-t/5} = rhs. \quad \checkmark$$

Also, if $v(t) \equiv 49$

then $lhs = v'(t) \equiv 0$ and $rhs = \frac{1}{5}\left[49 - 49\right] = 0.$

A particular soln can be obtained by given an additional condition such as

$$\boxed{v(0) = 0} \qquad (3)$$

Combining (2) and (3).

$$0 = v(0) = C_2 + 49 \Rightarrow C_2 = -49.$$

and

$$\boxed{v(t) = 49\left(-e^{-t/5} + 1\right).} \qquad (4)$$

Show this Soln. in MAPLE (direction field).

This Soln. is called a particular Solution.

The Set of equations:

$$\begin{cases} v'(t) = -\frac{1}{5}\,(v(t) - 49) & (5) \\ v(0) = 0 & (6) \end{cases}$$

is called an <u>initial value problem.</u>

Now, let's consider the IVP:

$$\begin{cases} v'(t) = -\frac{1}{5}\,[v(t) - 49] \\ v(0) = v_0 \end{cases}$$

By replacing (6) into (2)

$$v_0 = v(0) = C_2 + 49 \Rightarrow C_2 = v_0 - 49$$

Thus, $\boxed{v(t) = (v_0 - 49)\, e^{-t/5} + 49}$ (7)

or $v(t) = 49\left(1 - e^{-t/5}\right) + v_0 e^{-t/5}$

We will show later that (7) contains all possible solns. of (1) (for any v_0) and it is called the <u>General Soln. of (1)</u>