2.2. Separable Equations.

Consider

$$\frac{dy}{dx}(x) = \frac{-x}{y(x)} \qquad (1.1)$$

Non-linear.

It can be rewritten as

$$y(x)\,\frac{dy}{dx}(x) = -x$$

Integrating w.r.t. x : $\int dx$

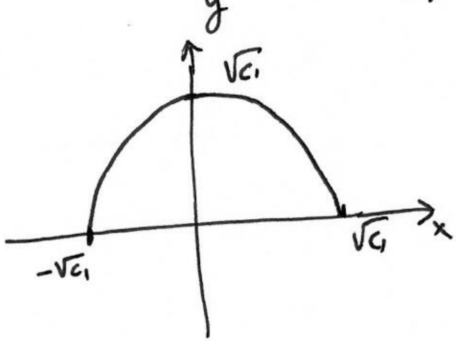
$$\frac{y^2(x)}{2} = -\int x\,dx = -\frac{x^2}{2} + C$$

or

$$y(x) = \pm\sqrt{C_1 - x^2}$$

If $C>0$, the soln. corresponds to a half circle: of radius $\sqrt{C_1}$.

(1.2) $\boxed{y^2 + x^2 = C_1}$



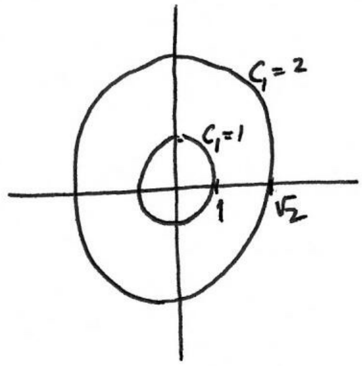
An alternative and practical way to arrive to the same solution is to write (1.1) in "differential form"

$$y\,dy = -x\,dx$$

Integrating $\int dy \quad \int dx$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

or $$\boxed{x^2 + y^2 = 2C = C_1}$$

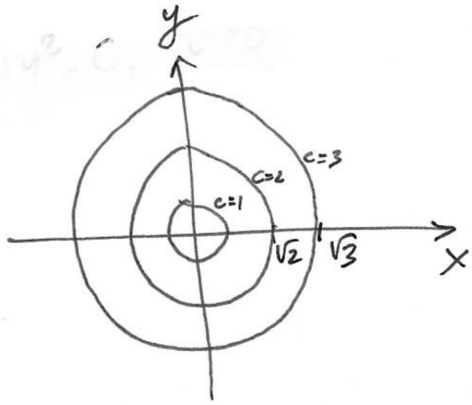


This family of curves is called "Integral curves"

The graphs of solutions of (1.1) usually are part of the integral curves.

Integral curves: $x^2+y^2=c$

Graph of solution usually is part of integral curve.



Exercise 22.-

$$y'(x) = \frac{3x^2}{3y^2(x)-4}, \quad y(1)=0$$

Singular at $y = \pm \frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$

In differential form:

$$(3y^2-4)\,dy = 3x^2\,dx$$

Integrating both sides:

$$y^3-4y = x^3+C \Rightarrow \boxed{y^3-4y-x^3=C}$$

family of integral curves.

Using I.C.

$$0 = 1+C \Rightarrow C=-1$$

$$\Rightarrow \boxed{y^3-4y-x^3=-1}$$ particular integral curve.

Go TO MAPLE for graphing and analyzing behavior of solutions.

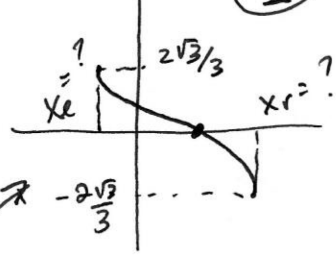
Soln. to IVP: $y' = \dfrac{3x^2}{3y^2-4}$, $y(1)=0$.

$$y^3 - 4y - x^3 = -1 \qquad (*)$$

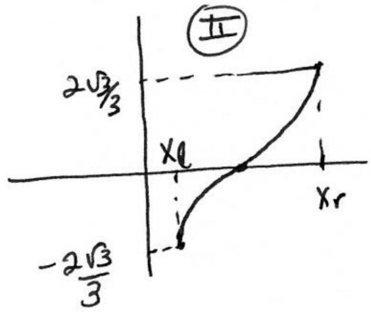
Singularity at $y = \pm\frac{2\sqrt{3}}{3}$.

Several possibilities:

I



II



Interval of Validity : (x_ℓ, x_r).

Subst. into $(*)$ $\left(\frac{2\sqrt{3}}{3}\right)^3 - 4\left(\frac{2\sqrt{3}}{3}\right) - x^3 = -1 \Rightarrow x \simeq -1.2763\ldots = x_\ell$

also $\left(-\frac{2\sqrt{3}}{3}\right)^3 - 4\left(-\frac{2\sqrt{3}}{3}\right) - x^3 = -1 \Rightarrow x \simeq 1.5978\ldots = x_r$

So from analysis (Not graphic)

Soln : $y^3 - 4y - x^3 = -1$, $\quad -1.276 < x < 1.5978$.

interval of validity.

Example 3.- Non-Separable.

$$\left[2x^2 + y(x)\right] \frac{dy}{dx}(x) = 2x$$

$$2x^2 y'(x) + \underbrace{y(x)\, y'(x)}_{\text{non-linear}} = 2x$$

In general,

$$\frac{dy}{dx}(x) = f(x,y) \qquad (*)$$

If $\quad f(x,y) = \dfrac{M(x)}{N(y)}$

then $\quad \dfrac{dy}{dx}(x) = \dfrac{M(x)}{N(y(x))}$

and the variables can be separated

$$N(y(x))\, \frac{dy}{dx}(x) = M(x)$$