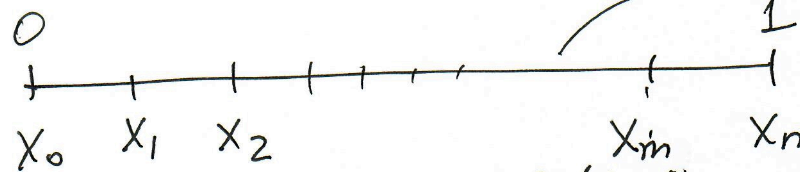
Nonhomogeneous Dirichlet

Solve

$$\begin{cases} -u''(x) + u(x) = x, & 0 < x < 1 \\ u(0) = 0, \quad u(1) = \beta \end{cases}$$

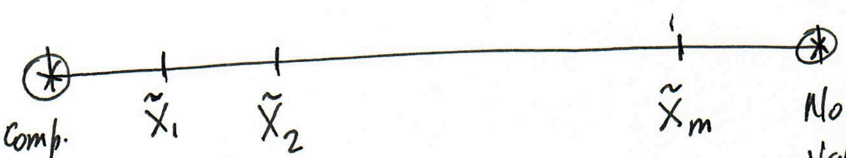
First, define a grid:



$h = 1/m+1$

In paper. and book

$m_1 + 2$ grid points.



Dirichlet problem

No comp. Value is assigned

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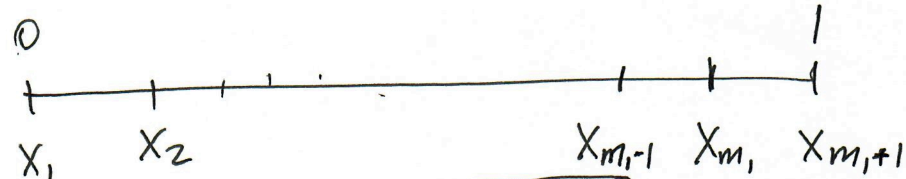
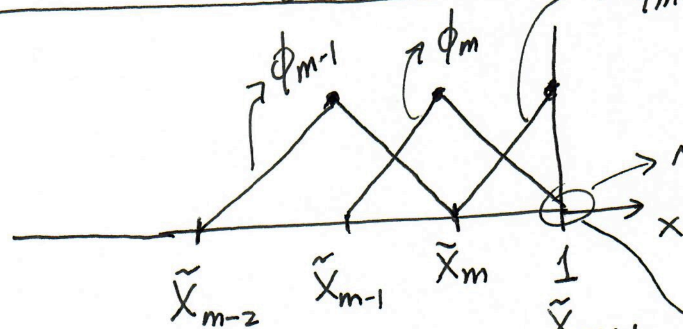
In the Leveque codes:



$h = \frac{1}{m_1}$

$\boxed{m = m_1 - 1}$ interior points.

Dirichlet Cond. at right end:



No computation Assign BC. value.

$u(\tilde{x}_{m+1}) = \beta$

Variational form with Dirichlet BCs.

$$\int_0^1 (u''v - uv)\,dx = -\int xv\,dx$$

$$\int_0^1 u''v\,dx - \int uv\,dx = -\int xv\,dx$$

$$u'v\Big|_0^1 - \int_0^1 u'v'\,dx - \int_0^1 uv\,dx = -\int_0^1 xv\,dx.$$

or $$\int_0^1 u'v'\,dx + \int_0^1 uv\,dx = u'(1)v(1) - u'(0)v(0) + \int_0^1 xv\,dx$$

Assuming $v(0) = 0 = v(1)$.

$$\int_0^1 u'v'\,dx + \int_0^1 uv\,dx = \int_0^1 xv\,dx.$$

$$\boxed{a(u,v) = (x,v)}.$$

If BVP is Dirichlet nonhomogeneous.

$u(0) = 0, \quad u(1) = \beta \neq 0.$

We maintain the condition $v(0) = 0 = v(1)$

and express the solution $u(x)$ as

$$\boxed{u(x) = \sum_{j=1}^{m} u_j \phi_j + \overset{=\beta}{u_{m+1}}\,\phi_{m+1}}$$

Soln $u(x)$ in terms of basis functions:

$$\boxed{u(x) = \sum_{j=1}^{m}{}' u_j \phi_j + \overset{=\beta}{u_{m+1}} \phi_{m+1}} \quad \text{(NH3)}$$

Discrete equation at $x = \tilde{x}_m$

$$\boxed{a(u, \phi_m) = (f, \phi_m)} \quad \text{(N.H.4)}$$

Cut h...!

Substituting (NH.3) into (NH.4), we obtain the last equation for the matrix at $\underline{x = x_m}$

Find it!