Integration Review

Fall 04

(a) The integral $\int_0^{\pi/2} \sin x \, dx$ equals _____

(b) The integral $\int \cos x \sin^2 x \, dx$ equals _____

(c) The integral $\int_0^{\pi/2} \sin^2 x \, dx$ equals _____

(d) The integral $\int_0^{\pi/4} \sec^2 x \, dx$ equals _____

(e) The integral $\int \tan x \, dx$ equals _____

(f) The integral $\int \frac{x}{\sqrt{1-x^2}} \, dx$ equals _____

(g) The integral $\int \frac{e^{1/x}}{x^2} \, dx$ equals _____

(h) The integral $\int_0^{\infty} \frac{dx}{1+x^2}$ equals _____

(i) The integral $\int_0^{1} \frac{dx}{\sqrt{1-x^2}}$ equals _____

(j) The integral $\int_1^{2/\sqrt{3}} \frac{dx}{x\sqrt{x^2-1}}$ equals _____

(k) The integral $\int_0^{1} \frac{dx}{x^2}$ equals _____

(l) The integral $\int_1^{\infty} \frac{dx}{x^2}$ equals _____

Fall 05

(a) The integral $\int \cos(x+2) \, dx$ equals _____

(b) The integral $\int \sec x \tan x \, dx$ equals _____

(c) The integral $\int_0^{1} \frac{dx}{1+x^2}$ equals _____

(d) The integral $\int_0^{1} \frac{dx}{\sqrt{1-x^2}}$ equals _____

(e) The integral $\int \tan^2 x \, dx$ equals _____

(f) The integral $\int_0^{1} \frac{dx}{\sqrt{x}}$ equals _____

(g) The integral $\int_0^\infty \frac{dx}{x^3}$ equals _____

(h) The integral $\int \frac{x}{\sqrt{1+x^2}}\,dx$ equals _____

(j) State the integration by parts formula:

_____

(k) Give a limit definition of the improper integral $\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$

_____

(m) The integral $\int \cot x\,dx$ equals _____

Winter 06 2. $\int_0^{\pi/2} \sin x\,dx =$ _____

3. $\int \tan x \sec^2 x\,dx =$ _____

5. $\int \frac{x^2}{\sqrt{x^2+4}}\,dx$ can be integrated using the trigonometric substitution $x =$ _____

Fall 06 (a) Does the improper integral $\int_0^\infty \frac{dx}{e^x+1}$ converge (yes or no) _____

(b) The integral $\int \frac{\cos x}{\sin^3 x}\,dx$ equals _____

(c) The integral $\int_1^{e^2} \frac{dx}{2x}$ equals _____

(f) If $n > 1$, the integral $\int_1^\infty \frac{dx}{x^n}$ equals _____

(h) The integral $\int x \sin x\,dx$ equals _____

Fall 07 (g.) The integral $\int_0^\infty \frac{dx}{1+x^2}$ equals_____

(h.) Here is an antiderivative: $\int \frac{\sqrt{9-x^2}}{x^2}dx$. Tell what substitution to use in order to find this antiderivative._____

(i.) The integral $\int_0^1 \frac{1}{x^{2/3}}dx$ equals _____

(j.) The antiderivative $\int x\sin(x)\,dx$ equals_____

Fall 04 2. Find $\int_1^2 x^3\ln(5x)\,dx$

(a) $4\ln 2+\frac{15}{4}\ln 5-\frac{15}{17}$

(b) $4\ln 2+\frac{15}{4}\ln 5-\frac{15}{16}$

(c) $4\ln 2+\frac{3}{4}\ln 5-\frac{15}{16}$

(d) $4\ln 2+\frac{11}{4}\ln 5-\frac{15}{16}$

(e) $4\ln 3+\frac{15}{4}\ln 5-\frac{15}{16}$

(f) $2\ln 2+\frac{15}{4}\ln 5-\frac{15}{16}$

(g) None of the above

4. Find $\int_0^1 x^2\sin(2\pi x)\,dx$.

(a) 2π

(b) π^2

(c) $-\frac{1}{2\pi}$

(d) 0

(e) $\frac{-1}{2\pi}+\frac{1}{2\pi^2}$

(f) $\frac{1}{2\pi}$

(g) $\frac{1}{2\pi^2}$

(h) $\pi^2-\frac{1}{2\pi}$

(i) $\frac{1}{2}$

(j) None of the above

F05 3. The most appropriate first step to integrate $\int \frac{x^2-1}{3x^3-x^2}\,dx$ would be

(a) Integration by parts

(b) Partial fractions

(c) Trigonometric Substitution

(d) Other (non trigonometric) substitution

(e) Differentiate the integrand

(f) None of these

5. The improper integral $\int_0^\infty xe^{-x}dx$ converges to

(a) 0 (e) 2

(b) $1/e$ (f) e

(c) 1/2 (g) None of these

(d) 1 (h) It doesn't converge

W06 7. $\int_1^\infty \frac{1}{x^p}\,dx$

(a) converges if $0 < p < 1$ (b) converges if $p = 1$ (c) converges if $p > 1$

(d) diverges if $p > 1$ (e) diverges if $p > 0$ (f) diverges if $p \neq 1$

(g) none of these

9. The value of $\int_{-a}^{a} \sqrt{a^2 - x^2}\,dx$ is

(a) $\frac{1}{2}a$ (b) $\frac{1}{4}a^2$ (c) a (d) πa^2 (e) $\frac{1}{2}\pi a^2$ (f) $\frac{1}{4}\pi a^2$

(g) $\frac{1}{2}\pi^2$ (h) $\frac{1}{4}\pi^2 a$ (i) $-\frac{1}{2}\pi a^2$ (j) none of these

13. $\int e^x \cos x\,dx =$

(a) $e^x \sin x + C$ (b) $e^x \cos x - e^x \sin x + C$ (c) $e^x \sin x - e^x \sin 2x + C$

(d) $\frac{1}{2}e^x \sin x + \frac{1}{2}e^x \cos x + C$ (e) $2e^x \sin x - 2e^x \cos x + C$ (f) none of the above

F06 3. Which of the following substitutions will best simplify the integral $\int \sqrt{3 + 2x - x^2}\,dx$?

(a) $x = 1 - 2\sec u$ (e) $x = \sqrt{3}\sin u$

(b) $x = \sqrt{3} + 2\cosh u$ (f) $x = 1 + 2\sin u$

(c) $x = \sqrt{3}\cos u$ (g) $x = 2\sin u$

(d) $x = \sqrt{3} - 2\cosh u$

7. The integral $\int_2^{e+1} (x-1)\ln(x-1)\,dx$ is equal to

(a) $\frac{e^2-1}{2}$ (d) $\frac{e^2+1}{4}$

(b) e^2+1 (e) $\frac{e^2-1}{4}$

(c) $\frac{e^2+1}{2}$ (f) e^2-1

FD4 10. Evaluate the definite integral $\int_0^1 x^3\sqrt{1-x^2}\,dx$.

11. (a) Evaluate the indefinite integral $\int \left(\frac{3x}{2+3x^2} + \frac{x}{3-x}\right) dx$

(b) Derive the partial fraction expansion for $\frac{3x^2-x-2}{(x+1)(x^2+1)}$.

FD5 10. (a) Evaluate the integral $\int_0^1 t^2 e^t\,dt$.

(b) Expand in partial fraction form $\frac{x^2+3}{x^2-1}$.

(c) Evaluate the integral $\int \frac{x^2+3}{x^2-1}\,dx$.

11. Evaluate the integral $\int \frac{1}{4-3\sin x}\,dx$.

W06 14. Find a formula for $\int \sqrt{1-a^2x^2}\,dx$ $(a>0)$

16. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$.

18. Find $\int \frac{x+1}{x^2-4}\,dx$.

19. Determine whether each integral converges, and give its value if it does so.

(a) $\int_0^\infty x^2 e^{-x^3}\,dx$

(b) $\int_{-1}^1 \frac{1}{\sqrt{|x|}}\,dx$

F06 9. Evaluate each integral

(a) $\int \frac{dx}{2+x-x^2}$

(b) $\int \sec^3(2x)\,dx$

18. Find the definite integral $\int_0^1 x^3\sqrt{1-x^2}\,dx$.

F07 16 Find the following

a (6 points) $\int_0^{\pi/2} \sin^3(x)\cos^3(x)\,dx$

b (6 points) $\int \frac{x+5}{x^2-1}dx$.

c (6 points) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$